Time Horizons And Emissions Trading

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Abstract

This paper studies cap and trade schemes where feedback mechanisms from the market for allowances determine the cap on emissions and where the scheme ends in a final period T. The effect of bringing forward the final period from T to $\bar{T} < T$ depends on the supply mechanism in place: under a price mechanism (supply increasing in the allowance price) the reduction in equilibrium emissions is positive and bounded from below, whereas it is bounded from above and possibly negative under a quantity mechanism (supply decreasing in the surplus of unused allowances). I characterize these bounds and provide conditions under which they coincide. I also identify sufficient conditions for which an earlier final period leads to strictly higher emissions under a quantity mechanism.

Keywords: Emissions trading, market-based emissions regulations, policy design
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1 Introduction

Policies to control externalities often rely on markets. Market-based regulation, it is believed, allows agents covered by the policy to achieve a given objective at least cost. Moreover, conditions prevailing in the market signal valuable information that can be used to update the policy and better align its parameters with economic efficiency. Yet any market is a place of many signals and though in theory a policymaker could use all of those, often only a subset is used. The question then arises whether and, if so, to what extent the type of information used for policy-updating matters. This paper addresses that issue by comparing two general classes of updating policies, price and quantity mechanisms, in the context of regulations with an explicitly dynamic component. I show that policies using information on prices combine more naturally with regulations directly governing the timing of externalities than do policies using information on quatities. I henceforth use the language of emissions trading for narrative convenience though the economic forces that drive my results are general and extend beyond that application.

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In its most basic form, emissions trading, or cap and trade, sets a cap on emissions but lets polluters freely trade emissions under the cap. For various reasons, economists and policymakers advocate making the cap endogenous to conditions prevailing in the market thus created. Such policies generally come in two forms: price and quantity mechanisms. Under a price mechanism, the cap is increasing in the allowance price. Price mechanisms were proposed by Roberts and Spence (1976), Pizer (2002), and Abrell and Rausch (2017); examples in practice are price collares, used in California's ETS (Borenstein et al., 2019). Under a quantity mechanism, the cap is tightened when the surplus of unused allowances increases. Quantity mechanisms were proposed by Kollenberg and Taschini (2016, 2019), Abrell and Rausch (2017), Lintunen and Kuusela (2018), Pizer and Prest (2020), and Quemin and Trotignon (2021). Examples in practice are abatement bounds (Holt and Shobe, 2016; Abrell and Rausch, 2017), a market stability reserve like the EU's (Perino, 2018; Gerlagh and Heijmans, 2019), or Korea ETS' liquidity provisions (Asian Development Bank, 2018).

Price and quantity mechanisms are intuitively similar. A low price or a large surplus are interpreted to indicate that abatement is cheap, motivating a tightening of the emissions cap. Both mechanisms thus aim to better align the supply and demand for emissions. Apparant similarities notwithstanding, there is a fnudamental distinction between the mechanisms. A price mechanism uses prices to update quantities, turning the quantity-instrument that is cap and trade into a hybrid policy. A quantity mechanism instead uses quantities to update quantities – it doubles down on the quantity aspect of emissions trading.

This paper compares price and quantity mechanisms. Consider a cap and trade scheme in which the cap on emissions is set through either a price or a quantity mechanism. The scheme ends in a final period which determines the time horizon of emissions tradig. I show that the reduction in equilibrium emissions when the final period is advanced from T to $\bar{T} < T$ is positive and bounded from below under a price mechanism, whereas it is bounded from above and possibly negative under a quantity mechanism. I characterize these bounds and identify sufficient conditions under which they coincide, that is, under which the a price mechanisms outperforms a quantity mechanism. I also formulate sufficient conditions for an earlier final period to cause strictly higher emissions under a quantity mechanism. My results establish that price and quantity mechanisms are far from equivalent. Price mechanisms are more easily combined with policies that directly try to control the timing of emissions.

The driving force behind my results is firms' incentive not to hold any allowances once the final period arrives. An allowance has value only if it can be used to cover emissions. Rather than leave allowances unused until the scheme ends, firms use them before the final period to lower abatement costs while they can. A shorter time horizon hence incentivzes firms to use more allowances early on, reducing the bank of allowances and putting downward pressure on the allowance price. Because of the latter effect, a price mechanism – if anything – reinforces the effect on an earlier emissions ban. A quantity mechanism in contrast increases the early-period supply of allowances due to the reduction in banking, undoing part or all of the emissions cuts achieved due to an earlier ban.

Naturally, the policymaker could remedy the adverse supply effec under a quantity mechanism by accompanying the (earlier) final period with a 'manual' reduction of supply. Such a solution raises several practical questions, including how many allowances should be taken out of the market in this way and when exactly they ought to be removed. More fundamentally, the need for an ad hoc solution indicates a benefit of price over quantity mechanisms: whereas a quantity mechanism can be made to work only after additional measures are taken, a price mechanism takes care of itself.

I derive my results under general assumptions about abatement costs and supply mechanisms. Abatement costs should be convex and increasing, whereas for price and quantity mechanisms only the signs of the first derivatives are restricted. The sufficiency of such minimal assumptions, rather than specific functional forms, indicates a deep-rooted advantage of price over quantity mechanisms. The choice of generality over narrower parametric specifications naturally impedes a welfare analysis. Importantly, though, my results do not concern welfare per se but rather the internal consistency of a set of policies; they show that specific policies cannot be combined in a straightforward way. To the extent that policies are chosen with specific welfare goals in mind, the present analysis shows that these goals can be unattainable.

The final period that ends the scheme has various interpretations. First, the scheme might simply end. An example would be situations in which the policymaker aims to eliminate pollution over time and regulates the transitionary period using cap and trade. Second, the final period could be a consequence of a seperate ban on emissions. In this interpretation, there is no end to the scheme per se; rather, there is another policy, independent of the scheme, that bans emissions. Such bans can come in two ways, hard and soft. A hard ban forbids emissions entirely while a soft ban merely requires that any amount of emissions be compensated for by an equal amount of negative emissions. Future bans on emissions, whether hard or soft, are rapidly gaining prominence as a climate policy instrument (Rogelj et al., 2021; Höhne et al., 2021). Third, the final period could be implied by a policy of retiring emissions allowances. Policymakers often do not intend to keep on supplying new allowances indefinitely. If so, a policy that retires unused allowances after L years effectively implies an end to the cap and trade scheme exactly L years after allowences are last supplied.

A particular pollution problem to which this analysis applies is climate change. To mitigate global warming, policymakers often rely on a triplet of climate policies: emissions trading, price or quantity mechanisms, and a future ban on emissions. The first, emissions trading for greenhouse gases, is used in more than 30 jurisdictions representing 54% of global GDP (ICAP, 2021). Still more are under way or at least considered. The second, price or quantity mechanisms, complement a large majority of those cap and trade schemes.[1] They have a major impact on existing emissions trading schemes; for example, Borenstein et al. (2019) show that the allowance price in California's cap and trade scheme is determined by the administrative price collar with 98.9 percent probability. The third, a final period on emissions, reflects the wave of zero emission targets that governments have recently pledged to and which are considered crucial in meeting the Paris Agreement temperature goals (Höhne et al., 2021).

The focus on the timing of emissions notwithstanding, this paper has broader implications. Any policy intended to reduce future emissions exercises a downward pressure on banking incentives and the allowance price. Price mechanisms hence reinforce such policies by supplying fewer allowances while quantity mechanisms counteract them by loosening the cap

[1]While not extensive, a list of cap and trade schemes that operate price or quantity mechanisms of the kind studied in this paper includes California's cap and trade scheme, China's National ETS, the EU ETS, Germany's National ETS, Korea's ETS, New Zealand's ETS, the Massachusetts Limits on Emissions from Electricity Generator, the Regional Greenhouse Gas Initiative (RGGI), and Switzerland's ETS.

on emissions. I chose the extreme case of a final period on emissions for largely pragmatic reasons. First, as I argue above, a final period on emissions actually appears to be on the global policy table (Höhne et al., 2021). Second, a final period facilitates precise characterization of the upper/lower bound on emissions reductions. In a narrow sense, this paper warns that future bans on emissions do not easily combine with cap and trade schemes that govern supply through a quantity mechanism. More broadly, it suggests that quantity mechanisms are generally harder to combine with other policies. Either way, price mechanisms have a clear edge over quantity mechanisms.

The paper proceeds as follows. Section 2 sets out the model and defines price and quantity mechanisms formally. Section 3 gives important comparative statics results for the analysis. Section 4 defines the equilibrium in cap and trade schemes under price and quantity mechanisms. Section 5 presents the main results, and Section 6 concludes.

2 Model

2.1 Building blocks

Consider a dynamic market consisting of a set $N = \{1, 2, ..., n\}$ of polluters, $n > 1$, called firms for simplicity. In each period $t \geq 0$, abatement for firm i is given by $a_{it} = \bar{q}_{it} - q_{it}$, where $\bar{q}_{it}$ denotes business-as-usual emissions (i.e. the level emissions in the absence of any policy whatsoever) and $q_{it} \geq 0$ is the actual level of emissions in period t. The cost of abatement is determined by the abatement cost function C_{it} which satisfies $C_{it}(0) = 0$, $C'_{it}(a_{it}) := \frac{\partial C_{it}(a_{it}}{\partial a_{it}} > 0$, and $\frac{\partial^2 C_{it}(a_{it})}{\partial a_{it}^2} \geq 0$. To simplify the exposition and reduce notation, I assume perfect foresight about C_{it} throughout the main text. This assumption is harsh but inconsequential: Appendix B establishes that all results generalize (in expectations) to environments characterized by uncertainty about future abatement costs.

Emissions are regulated through a cap and trade scheme. Let s_{it} denote the number of allowances supplied to firm i at the start of period t. Allowances are tradeable on a secondary market where a firm can sell or acquire them at a price p_t which it takes as given.[2] I assume that every allowance bought must also be solds, so $\sum_i m_{it} = 0$ for all t.[3] Hence, if a firm chooses an amount q_{it} of emissions and buys a total of m_{it} allowances on the secondary market, its total costs in period t are $C_{it}(\bar{q}_{it} - q_{it}) + p_t m_{it}$.

Emissions may not, in principle, exceed the supply of allowances. Temporal violations of the periodic cap s_{it} are facilitated through a banking provision (Kling and Rubin, 1997; Hasegawa and Salant, 2014). I define banking by firm i in period t to be $b_{it} := s_{it} + m_{it} - q_{it}$. The bank of allowances held by firm i at the start of period t is therefore $B_{it} := \sum_{s=0}^{t-1} b_{is} = B_{it-1} + b_{it-1} = B_{it-1} + s_{it-1} + m_{it-1} - q_{it-1}$, and the total bank of allowances at the start of period t is $B_t := \sum_i B_{it}$. I also assume that borrowing is not allowed, so $B_{it} \geq 0$ for all i and

[2] Though the assumption of price taking behavior is strong, it is also standard in the literature on emissions trading (*c.f.* Pizer, 2002; Hasegawa and Salant, 2014; Holland and Yates, 2015; Abrell and Rausch, 2017; Pizer and Prest, 2020; Holtsmark and Midttømme, 2021). Notable exceptions that study the cap and trade when (some) firms can influence the market are Liski and Montero (2011) and Harstad and Eskeland (2010).

[3] This constraint is not entirely obvious. It could be violated if the policymaker, as Holland and Yates (2015) propose, were to put an "exchange rate" on allowances. Because such exchange rates are not common in practice, I stick with constraint (3).

t; this assumption is not necessary, but it is realistic. The *dynamic* constraint on emissions by firm i is hence $\sum_{s=0}^{t} q_{is} \leq \sum_{s=0}^{t} s_{is} + m_{is}$ for all t.

The scheme ends with a final period $T > 0$; that is, $q_t = 0$ for all i and all $t \geq T$. The final period is a hard constraint in the sense that unused allowances cannot be used to cover emissions in or after T. The special case in which the scheme never ends is obtained by letting $T \to \infty$. I define $t = 0$ as the point in time starting from which all firms T is common knowledge. I colloquially refer to T as the *time horizon of emissions*. One is free to interpret the final period in multiple ways such as a plain end for the cap and trade scheme, a separate ban on emissions, or a policy of retiring allowances. I henceforth use the language of a ban on emissions.

2.2 Firms' problem

The elements of the model combined yield the following problem for the firm:

$$\begin{aligned}
\min_{q_{it}, m_{it}} \quad & \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[C_{i\tau}(\bar{q}_{i\tau} - q_{i\tau}) + p_\tau m_{i\tau} \right] && (1)\\
\text{subject to} \quad & \sum_{\tau=0}^{t} q_{i\tau} \leq \sum_{\tau=0}^{t} \left[s_{i\tau} + m_{i\tau} \right], && (2)\\
& \sum_{i} m_{it} = 0, && (3)\\
& q_{i\tau} = 0, \quad \text{for all} \quad \tau \geq T, && (4)\\
& B_{it+1} = B_{it} + s_{it} + m_{it} - q_{it}, && (5)\\
& B_{it} \geq 0, && (6)
\end{aligned}$$

for all $i \in N$ and $t \geq 0$ and where $\beta > 0$ is the discount factor.

The first-order conditions to this problem are standard and given in the Appendix. For each i and t, let $q_{it}(p_t, T)$ denote the level of emissions that solves the constrained optimization problem above. Convexity of C_{it} implies:

$$\frac{\partial q_{it}(p_t, T)}{\partial p_t} < 0, \tag{7}$$

for all $t < T$. As is intuitive, the abatement cost minimizing level of emissions chosen by firm i in period t is decreasing in the prevailing allowance price in that period.[4] For given p_t and T, define $q_t(p_t, T) = \sum_i q_{it}(p_t, T)$ to be total demand for emissions in period t.

Lemma 1. *For all $t \in \{0, ..., T-1\}$, aggregate demand for emissions $q_t(p_t, T)$ is decreasing in the allowance price p_t.*

Proof.

$$\frac{\partial q_t(p_t, T)}{\partial p_t} = \sum_i \left[\frac{\partial q_{it}(p_t, T)}{\partial p_t} \right] < 0,$$

[4] There is the implicit assumption of a market economy here. As Cao et al. (2021) document for China's pilot ETS, when regulated sectors are strictly regulated by the government, firms may not have the freedom to trade until marginal abatement costs (roughly) equal the allowance price.

where the inequality is an immediate implication of (7). $\square$

Lemma 2. *For all $t \in \{0, ..., T-1\}$, cost-minimizing prices co-move between periods:*

$$\frac{\partial p_{t+1}}{\partial p_t} > 0. \tag{8}$$

Lemma 2 gives a differential version of Hotelling's rule. It is well known that factors such as asymmetric information (Martimort et al., 2018), arbitrage (Anderson et al., 2018), or rolling horizons (Spiro, 2014) cause violations of the rule in its canonical formulation. The literature nevertheless supports the co-movement of prices over time predicted by Lemma.

Lemmas 1 and 2 hold for $t < T$. The following remarks explain.

Remark 1. *$m_{it} = 0$ for all i and $t \geq T$. That is, firms neither buy nor sell allowances on the secondary market from T onward.*

Remark 2. *The allowance price p_t need not satisfy* (8) *when $t \geq T$. The allowance price comes about through supply and demand on the secondary market. As there is no demand for allowances when $t \geq T$, the allowance price is not properly defined for those periods.*

2.3 Supply mechanisms

Let $s_t = \sum_i s_{it}$ denote the total supply of allowances in period t. I will come to the precise determination of the supply path (s_t) shortly; in any case I assume that $\sum_{s=0}^{t} s_s < \sum_{s=0}^{t} \bar{q}_s$ for all t, where $\bar{q}_t = \sum_i \bar{q}_{it}$. That is, the supply of allowances does not exceed business-as-usual emissions.[5]

The first class of supply mechanisms considered are price mechanisms. To avoid confusion, the supply of allowances under a price mechanism is denoted s_t^P.

Definition 1 (Price mechanism)**.** *A cap and trade scheme operates a* price mechanism *if the supply of allowances in any period t is weakly increasing in the prevailing allowance price p_t. Formally, for any period t and any two price levels p_t and p_t' it holds that $s_t^P(p_t) > s_t^P(p_t')$ if and only if $p_t > p_t'$.*

Price mechanisms were proposed by Roberts and Spence (1976), Pizer (2002), and Abrell and Rausch (2017).[6] Practical examples are price collars (Borenstein et al., 2019). I assume that $s_t^P(0) \leq q_t(0, T)$ and $s_t^P(\infty) \geq q_t(\infty, T)$ for all t, with a stict inequality for at least one t. While not strictly necessary for my main results, I assume that $s_t^P(p_t)$ is differentiable in p_t to simplify the exposition. I write B_t^P fpor the bank of allowances when supply is governed by a price mechanism.

Given a price vector $p = (p_t)$, let the total supply of allowances under a price mechanism between any two periods t_1 and t_2 be denoted $S^P(t_1, t_2 \mid p)$:

$$S^P(t_1, t_2 \mid p) = \sum_{t=t_1}^{t_2} s_t^P(p_t). \tag{9}$$

[5]The case in which allowance supply exceeds BAU emissions appears empirically irrelevant (Fowlie, 2010; Fowlie et al., 2012; Deschenes et al., 2017; Calel, 2020; Bayer and Aklin, 2020).

[6]Ambec and Coria (2021) study the use of an emissions tax to gather information that can be used to set an emissions standard. Such a policy is de facto similar to a price mechanism.

The second class of supply mchanism studied are quantity mechanisms. Let the supply of allowances under a quantity mechanism be denoted s_t^Q.

Definition 2 (Quantity mechanism)**.** *A cap and trade scheme operates a* quantity mechanism *if the supply of allowances in period t is increasing in the aggregate excess supply at the start of period t. That is, for any period t and any two B_t and B'_t, it holds that $s_t^Q(B_t) > s_t^Q(B'_t)$ if and only if $B'_t > B_t$.*

Quantity mechanisms were studied by Harstad and Eskeland (2010), Kollenberg and Taschini (2016, 2019), Abrell and Rausch (2017), Lintunen and Kuusela (2018), Pizer and Prest (2020), and Quemin and Trotignon (2021). Examples in practice are abatement bounds (Holt and Shobe, 2016; Abrell and Rausch, 2017), a market stability reserve like the EU's (Gerlagh et al., 2021), or Korea ETS' liquidity provisions (Asian Development Bank, 2018). I assume that $s_t^Q(B_t(p)) \leq q_t(0, T)$ and $s_t^Q(B_t(p)) \geq q_t(\infty, T)$ for all p, with a strict inequality for at least one t. While not strictly necessary for my main results, I assume that $s_t^Q(B_t)$ is differentiable in B_t. I also assume that $-1 < \partial s_t^Q / \partial B_t$ for all t to avoid the counter-intuitive scenario in which firms have an incentive to bank *less* today in order to have *more* allowances in the future. When supply is governed by a quantity mechanism I write B_t^Q for the bank of allowances.

Given a price vector $p = (p_t)$, let the total supply of allowances under a quantity mechanism between any two periods t_1 and t_2 be denoted $S^P(t_1, t_2 \mid p)$:

$$S^Q(t_1, t_2 \mid p) = \sum_{t=t_1}^{t_2} s_t^Q(B_t^Q(p)). \tag{10}$$

From the assumption that firms are price-takers follows that each firm, though cognizant of the supply mechanism in place, takes the supply of allowances as given. I also assume that the policymaker is committed to the supply mechanism in place. Policy commitment is a common assumption in the literature on emissions trading and supply mechanisms.

The timing of events is as follows. At the start of period t, the policymaker supplies s_t allowances according to the supply mechanism in place. Firms trade allowances on the secondary market and simultaneously choose their emissions q_t; unused allowances are banked. Markets clear and period $t + 1$ begins.

3 Dynamic price effects

Absent a supply mechanism, the effect of an increase in the allowance price on the bank of allowances is straightforward. A higher price p_t means that demand for emissions in period t goes down while leaving supply as is. This implies that the bank B_τ is increasing in p_t for all $\tau > t$. Moreover, since firms can freely bank allowances, the effect of an increase in p_t will be smoothed out over time, weakly increasing the bank of allowances in all periods $t \geq 1$. Though things are less straightforward when supply is governed by a price or quantity mechanism, the following two lemmas genealize this intuition.

Lemma 3 (Dynamic price effects under a price mechanism)**.** *Consider a cap and trade scheme that operates a price mechanism. Fix a final period on emissions T. For any two periods $\tau, t < T$, the bank of allowances B_t^P is increasing in the allowance price p_τ: $\frac{\partial B_t^P(p)}{\partial p_\tau} > 0$.*

Proof. Since $s_t(p_t)$ is increasing in p_t by construction while $q_t(p_t, T)$ is decreasing by (7), banking in period $b_t(p_t)$ is increasing in the allowance price p_t. Recall from (8) that prices co-move across periods. By implication, one has $\frac{\partial p_s}{\partial p_\tau} > 0$ for all $s, \tau \in \{0, 1, ..., T\}$ and therefore,

$$\begin{aligned} \frac{\partial B_t^P}{\partial p_\tau} &= \frac{\partial}{\partial p_\tau}\left[\sum_s^{t-1} s_s^P(p_s) - \sum_s^{t-1} q_s(p_s, T)\right] \\ &= \sum_s^{t-1} \frac{\partial s_s^P(p_s)}{\partial p_s}\frac{\partial p_s}{\partial p_\tau} - \sum_s^{t-1} \frac{\partial q_s(p_s, T)}{\partial p_s}\frac{\partial p_s}{\partial p_\tau} > 0. \end{aligned} \tag{11}$$

This establishes that B_t^P is increasing in p_τ for all $t, \tau \in [0, T)$. □

Lemma 4 (Dynamic price effects under a quantity mechanism)**.** *Consider a cap and trade scheme that operates a quantity mechanism. Fix a final period on emissions T. For any two periods $\tau, t < T$, the bank of allowances B_t is increasing in the allowance price p_τ: $\frac{\partial B_t^Q(p)}{\partial p_\tau} > 0$.*

Proof. The effect of an increase in the allowance price on first-period banking is straightforward:

$$\frac{\partial B_1^Q(p)}{\partial p_\tau} = \frac{\partial b_0(p_0)}{\partial p_0}\frac{\partial p_0}{\partial p_\tau} = \frac{\partial[s_0^Q - q_0(p_0, T)]}{\partial p_0}\frac{\partial p_0}{\partial p_\tau} = -\frac{\partial q_0(p_0, T)}{\partial p_0}\frac{\partial p_0}{\partial p_\tau} \geq 0, \tag{12}$$

where the inequality is strict for all p_0 such that $q_0(p_0, T) > 0$ and all $\tau \geq 0$. A little more work is required to determine the sign of $\partial B_t^Q / \partial p_\tau$ for $t > 1$. Recall that the bank of allowances evolves according to $B_t^Q(p) = B_{t-1}^Q(p) + s_{t-1}^Q(B_{t-1}^Q(p)) - q_{t-1}(p_{t-1})$, where s_t depends on B_t^Q because supply is governed by a quantity mechanism. Hence,

$$\frac{\partial B_t^Q(p)}{\partial p_\tau} = \frac{\partial B_{t-1}^Q(p)}{\partial p_\tau} + \frac{\partial s_{t-1}^Q(B_{t-1}^Q(p))}{\partial p_\tau} - \frac{\partial q_{t-1}(p_{t-1}, T)}{\partial p_\tau} \tag{13}$$

$$= \left(1 + \frac{\partial s_{t-1}^Q(B_{t-1}^Q(p))}{\partial B_{t-1}^Q(p)}\right)\frac{\partial B_{t-1}^Q(p)}{\partial p_\tau} - \frac{\partial q_{t-1}(p_{t-1}, T)}{\partial p_{t-1}}\frac{\partial p_{t-1}}{\partial p_\tau}. \tag{14}$$

The term in parentheses, $1 + \partial s_t^Q / \partial B_t^Q$, is positive by assumption. The final term in (14) is negative by (8) and (7). The only sign left to determine in (14) is hence that of $\partial B_{t-1}^Q / \partial p_\tau$; and this we know for $t = 2$. Using (12), induction on t establishes that

$$\frac{\partial B_t^Q(p)}{\partial p_\tau} \geq 0, \tag{15}$$

for all $t, \tau \in [0, T)$. The inequality is strict for all $p = (p_1, p_2, ...)$ that satisfy $q_t(p_t, T) > 0$ for at least one t. □

These lemmas have no immediate economic appeal but are useful when analyzing the time horizon of emissions on equilibrium outcomes. Such an analysis naturally requires a definition of equilibrium. The next section elaborates.

4 Equilibrium

The market is in equilibrium when the supply of emissions allowances is equal to demand subject to all policy constraints; prices adjust to brings about equilibrium. I define the equilibrium, for a given final period T, as a tuple (p, q, s) of a price vector $p = (p_t)$, an emissions vector $q = (q_t)$, and a supply vector $s = (s_t)$, for all $t \geq 0$. Equilibrium quantities are paths evaluated from the start of period 0 onward. In what follows, the term equilibrium emissions will refer to the sum total of emissions starting from period 0 (included) in equilibrium.

4.1 Price mechanisms

Consider a final period T. The market for allowances is in equilibrium if

$$\sum_{t=0}^{T} q_t(p_t^P, T) = \sum_{t=0}^{T} s_t^P(p_t^P), \tag{16}$$

where p_t^P is the equilibrium allowance price when the scheme operates a price mechanism and the final period on emissions is T. Let $p^P = (p_t^P)$ denote the associated equilibrium price vector. Observe that (16) can be rewritten as $B_T^P(p^P) = 0$, that is, in equilibrium firms have depleted the bank of allowances by the time emissions get banned.

An equilibrium is defined for arbitrary T. Consider then an alternative final period $\bar{T}$ and let $\bar{p}^P$ denote the associated equilibrium price vector. That is, $\bar{p}^P$ is the allowance supply path that solves (16) after substituting $\bar{T}$ for T.[7] Now define:

$$R^P(\bar{T}, T) := \sum_{t=0}^{T} s_t^P(p_t^P) - \sum_{t=0}^{\bar{T}} s_t^P(\bar{p}_t^P) = \sum_{t=0}^{T} q_t(p_t^P, T) - \sum_{t=0}^{\bar{T}} q_t(\bar{p}_t^P, \bar{T}). \tag{17}$$

In words, $R^P(T, \bar{T})$ is the reduction in equilibrium emisions (starting from period 0) when the final period is $\bar{T}$ rather than T, given that supply is governed by a price mechanism.

Under a price mechanism, the supply of allowances depends on the prevailing allowance price. Given a final period T, let $f^P(T)$ denote the period in which the supply of emissions drops permanently to zero under a price mechanism policy. Formally, $f^P(T) = \min\{t : s_\tau^P(p_\tau^P) = 0 \forall \tau \geq t\}$.

4.2 Quantity mechanisms

For a final period on emissions T, equilibrium under a quantity mechanism is reached when

$$\sum_{t=0}^{T} q_t(p_t^Q, T) = \sum_{t}^{T} s_t^Q(B_t^Q(p^Q)), \tag{18}$$

where p_t^Q is the competitive equilibrium allowance price in period t under a quantity mechanism and $p^Q = (p_t^Q)$. Observe that (18) can be rewritten as $B_T^Q(p^Q) = 0$, that is, in equilibrium firms have depleted the bank of allowances by the time emissions get banned.

[7] I.e. it solves $\sum_{t=0}^{\bar{T}} q_t(\bar{p}_t^P, \bar{T}) = \sum_{t=0}^{\bar{T}} s_t^P(\bar{p}_t^P)$, or $B_{\bar{T}}^P(\bar{p}^P) = 0$.

Equilibrium is defined for arbitrary T. Consider then an alternative final period $\bar{T}$ and let $\bar{p}^Q$ denote the associated equilibrium price vector. That is, $\bar{p}^Q$ is the allowance supply path that solves (18) after substituting $\bar{T}$ for T.[8] Now define:

$$R^Q(\bar{T},T) := \sum_{t=0}^{T} s_t^Q(B_t^Q(p^Q)) - \sum_{t=0}^{\bar{T}} s_t^Q(B_t^Q(\bar{p}^Q)) = q_t(p_t^Q,T) - q_t(\bar{a}_t^Q,\bar{T}). \tag{19}$$

In words, $R^Q(T,\bar{T})$ is the reduction in equilibrium emissions (starting from period 0) when the final period is $\bar{T}$ rather than T, given that supply is governed by a quantity mechanism.

Given a final period on emissions T, let $f^Q(T)$ denote the period in which the supply of allowances dries up permanently under a quantity mechanism. That is, $f^Q(T) = \min\{t : s_\tau^Q(B_\tau^Q) = 0 \forall \tau \geq t\}$.

At this point, the main question of this paper can be stated concisely as follows. Given two periods T and $\bar{T}$ such that $\bar{T} < T$, what are the properties of $R^P(\bar{T},T)$ and $R^Q(\bar{T},T)$?

5 Results

The thought experiment underlying this section, and therefore my results, is the following. Consider any two periods T and $\bar{T}$ such that $\bar{T} < T$. Suppose now that the regulator considers setting either T or $\bar{T}$ as the final period for the cap and trade scheme. How many emissions will be saved, in equilibrium, from having the final period at $\bar{T}$ rather than T?

5.1 Bounds

This section presents my most general results. I identify bounds on the reduction in equilibrium emissions from having th final period $\bar{T}$, rather than T, under price and quantity mechanisms.

5.1.1 Price mechanisms

Under a price mechanism, the reduction in equilibrium emissions from having the final period at $\bar{T}$, rather than T, is positive and bounded from below.

Proposition 1. *Consider a cap and trade scheme that operates a* price mechanism. *Consider two periods T and $\bar{T}$ such that $\bar{T} < T$. Let p^P denote the equilibrium allowance price vector when the final period is T. Then,*

$$R^P(\bar{T},T) \geq S^P(\bar{T},T \mid p^P) \geq 0. \tag{20}$$

That is, the reduction in equilibrium emissions in response to an earlier final period is positive and bounded from below under a price mechanism.

Ending the cap and trade schem earlier has two mutually reinforcing effects under a price mechanism. First, any allowances that would originally be supplied starting from period $\bar{T}$, $S^P(\bar{T},T \mid p^P)$, are taken out of the system. Second, firms redistribute their emissions to

[8] I.e. it solves $\sum_{t=0}^{\bar{T}} q_t(\bar{p}_t^Q,\bar{T}) = \sum_{t=0}^{\bar{T}} s_t(B_t^Q(\bar{p}^Q))$, or $B_{\bar{T}}^Q(\bar{p}^Q) = 0$.

early periods to avoid holding allowances once the scheme ends. Higher emissions in early periods suppress the allowance price in those periods. By the mechanics of a price mechanism, the lower price translates into a reduction of supply in the periods leading up to $\bar{T}$, further reducing emissions. While the first effect is always there, the second occurs only if firms originally hold a strictly positive bank of allowances at the start of period $\bar{T}$.

5.1.2 Quantity mechanisms

Under a quantity mechanism, the reduction in equilibrium emissions from having the final period at $\bar{T}$, rather than T, is bounded from above.

Proposition 2. *Consider a cap and trade scheme that operates a* quantity mechanism*. Consider two periods T and $\bar{T}$ such that $\bar{T} < T$. Let p^Q denote the equilibrium allowance price vector when the final period is T. Then,*

$$R^Q(\bar{T}, T) \leq S^Q(\bar{T}, T \mid p^Q). \tag{21}$$

That is, the reduction in equilibrium emissions in response to an earlier final period is bounded from above (and possibly negative) under a quantity mechanism.

Shortening the time horizon of emissions has two opposing effects under a quantity mechanism. First, any allowances that would originally be supplied starting from period $\bar{T}$, $S^Q(\bar{T}, T \mid p^Q)$, are eliminated. Second, firms may redistribute their emissions to early periods to avoid holding allowances by the time the final period arrives. These two effect are exactly the same for price and quantity mechanisms. However, the mechanics of a quantity mechanism dictate that a reduction in banking prior to $\bar{T}$ results in an *increase* in allowance supply in those periods. Under a quantity mechanism, the redistribution effect offsets some (or all) of the emissions reductions achieved by eliminating supply after period $\bar{T}$. The reduction in equilibrium emissions is therefore at most $S^Q(\bar{T}, T \mid p^Q)$, implying an upper bound.

Note that an earlier final period *may* reduce emissions under a quantity mechanism. It does not have to. In the next section, I provide sufficient conditions for the emissions reduction due to an earlier final period to be negative. Under these conditions, shortening the time horizon of emissions is incompatible with strengthened climate ambitions.

5.2 Incompatibility

Fix a final period T. Posit a period $T^* < T$ at which, given the final period T, equilibrium demand is strictly positive while equilibrium supply is already (and permanently) zero. There need not be such a T^* and if it exists it need not be unique. Assuming at least one exists, set $\bar{T} = T^*$. Formally, one can verify that the conditions on T and $\bar{T}$ thus imposed are:

$$q_{\bar{T}}(p^Q_{\bar{T}}, T) > 0, \tag{22}$$

and

$$f^Q(T) \leq \bar{T} < T. \tag{23}$$

If the policymaker advances the final period T to this $\bar{T}$, equilibrium emissions strictly increase.

Proposition 3. *Consider a cap and trade scheme that operates a* quantity mechanism*. Consider two periods T and $\bar{T}$. If T and $\bar{T}$ satisfy* (22) *and* (23)*, then*

$$R^Q(\bar{T}, T) < 0. \tag{24}$$

That is, equilibrium emissions are strictly higher *when the final period is $\bar{T}$ compared to when it is T.*

Conditions (22) and (23) have two implications. First, there is no supply of allowances after period $\bar{T}$ even when the final period is T. Hence, bringing forward the final period to $\bar{T}$ does not eliminate any supply between $\bar{T}$ and T, which by Proposition 2 implies that the reduction in equilibrium emissions is at most zero. Second, the fact that emissions used to be strictly positive in period $\bar{T}$, combined with the fact that supply reached zero already earlier, implies there must be a strictly positive bank of allowances in period $\bar{T}$ when the final period is T. Advancing the final period to $\bar{T}$ then triggers firms to deplete the bank earlier, implying less banking overall and therefore, under a quantity mechanism, more supply. As no supply is eliminated after period $\bar{T}$ while supply goes up before period $\bar{T}$, equilibrium emissions strictly increase when advancing the final period from T to $\bar{T}$.

5.3 Policy choice

Notwithstanding the foregoing, it is possible that emissions reductions under a quantity mechanism exceed those under a price mechanism; this could happen when the lower bound for a price mechanism lies strictly below the upper bound for a quantity mechanism. Here I argue that this possibility is somewhat contrived as it relies on asymmetries in baseline equilibrium allowance supplies.

To formalize this, fix a baseline final period T. Suppose that, given T, the equilibrium supply of allowances is the same under both a price and a quantity mechanism. Formally, given the baseline final period on emissions T, for all $t \geq 0$ let:

$$s_t^P(p_t^P) = s_t^Q(B_t^Q(p^Q)), \tag{25}$$

where p^P and p^Q again denote baseline equilibrium price vectors under a price and quantity mechanism, respectively. If (25) is satisfied, I say that the baseline equilibria under a price and quantity mechanism are *symmetric* under T. The next result shows that the lower and upper bound on emission reductions under a price and quantity mechanism, respectively, coincide when the baseline equilibria are symmetric in this sense.

Proposition 4. *Consider a baseline final period T and an earlier period $\bar{T}$ such that $\bar{T} < T$. If the baseline equilibria under a price and quantity mechanism are symmetric under T, then the reduction in equilibrium emissions when the final period is $\bar{T}$, compared to when it is T, is lower under a quantity mechanism than under a price mechanism:*

$$R^Q(\bar{T}, T) \leq R^P(\bar{T}, T). \tag{26}$$

For symmetric baseline equilibria, an earlier final period leads to higher emissions reductions under a price mechanism than under a quantity mechanism. Whereas the question of prices versus quantities is as old as environmental economics itself and depends on a score of factors (Weitzman, 1974; Hoel and Karp, 2001; Montero, 2002; Krysiak, 2008; Alesina and Passarelli, 2014), the choice between price and quantity mechanisms is much less ambiguous. Under comparable conditions, a price mechanism outperforms a quantity mechanism when it comes to achieving environmental ambitions.

6 Conclusions

I study cap and trade schemes where the cap on emissions is determined by conditions prevailing in the market for allowances. Attention is confined to two classes of empirically relevant supply policies.Under a price mechanism, the supply of allowances is increasing in the allowance price. Under a quantity mechanism, supply is reduced when the surplus of unused allowances increases. The scheme ends in a final period T, which the policymaker can choose freely. I show that the reduction in equilibrium emissions when the scheme ends in $\bar{T} < T$ rather than T is positive and bounded from below under a price mechanism, whereas it is bounded from above (and possibly negative) under a quantity mechanism. Precise characterization of these bounds are given. I identify sufficient conditions for which an earlier final period strictly increases emissions under a quantity mechanism. My results establish that price and quantity mechanisms are not equivalent and, depending on the broader policy environment, cannot be used interchangeably.

A natural qualification to the results on quantity mechanisms is the assumed exogeneity of the quantity mechanism to other policy changes. One might argue that a rational policymaker anticipates the effect of advancing the final period and could 'manually' reduce the supply of allowances accordingly. I concur. Even so, the clear benefit of price over quantity mechanisms remains: whereas a quantity mechanism can be made to work after additional measures are taken, a price mechanism takes care of itself.

Though not modeled explicitly, my results have implications beyond the narrow focus on the time horizon of emissions trading. Any policy intended to reduce emissions in the future exercises a downward pressure on banking incentives and the allowance price. A price mechanism hence reinforces such policies. A quantity mechanism instead counteracts the effect by issuing more allowances in early periods due to the reduction in banking. I chose the extreme case of a final period on emissions for largely pragmatic reasons. First, a hard final period facilitates precise characterizations of the upper/lower bounds on emissions reductions. Second, policies that directly target the timing and future banning of emissions are rapidly gainig popularity (Höhne et al., 2021; Rogelj et al., 2021).

In a sense, quantity mechanisms misinterpret market signals. They react to a reduction in banking as though it signaled an increase in the demand for emissions whereas, in reality, it is the response to a future (policy-driven) fallout of demand. This points to a more fundamental distinction between price and quantity information. While prices provide an accurate signal of the overall demand for emissions, quantities provide a signal only of *relative* demand, that is, of demand today relative to demand in the future. Being more efficient information aggregators, price signals are thus favored over quantity signals for market-based policy-updating.

This paper makes several restrictive assumptions which future work might seek to relax.

First, as discussed, I assume that firms are price takers. Though a common assumption in the literature, it is important to know to which extent the results presented here generalize to cases in which firms have market power (Harstad and Eskeland, 2010; Liski and Montero, 2011). Second, the focus on price and quantity mechanisms is somewhat restrictive as other kinds of supply mechanism, while uncommon in practice and the literature, could be conceived. Third, I discuss supply mechanisms in the context of a single emissions trading scheme. When multiple cap and trade schemes are linked (Holtsmark and Midttømme, 2021), different incentives may be at work; these are not considered here.

A Appendix

A.1 Firms' dynamic cost-minimization problem

Turning the constrained problem in (1)–(6) into an unconstrained cost minimization problem, each firm i chooses q_{it} and m_{it} to solve:[9]

$$\begin{aligned}\min_{q_{it},m_{it}} \sum_{t=0}^{T} \beta^t C_{it}(\bar{q}_{it} - q_{it}) + \sum_t \beta^t p_t m_{it} + \lambda_i \left[\sum_t q_{it} - s_{it} - m_{it}\right] + \sum_t \beta^t \mu_t \left[\sum_i m_{it}\right] \\ + \omega_{it}\left[B_{it} - B_{it-1} - s_{it-1} - m_{it-1} + q_{it-1}\right] + \beta^t \psi_{it} B_{it}.\end{aligned} \tag{27}$$

The first-order conditions associated with the cost-minimization problem given by (27) are:

$$-\beta^t C'_{it}(\bar{q}_{it} - q_{it}) + \lambda_i + \omega_{it+1} = 0, \tag{28}$$
$$\beta^t p_t - \lambda_i + \beta^t \mu_t - \omega_{it+1} = 0, \tag{29}$$
$$\omega_{it} - \omega_{it+1} + \beta^t \psi_{it} = 0. \tag{30}$$

Rewriting these first-order conditions gives:

$$C'_{it}(\bar{q}_{it} - q_{it}) + \psi_{it} = \beta C'_{it+1}(\bar{q}_{it+1} - q_{it+1}), \tag{31}$$

for all $t < T$. Moreover, each firm will emit, or abate, until marginal abatement costs roughly equal the allowance price,

$$p_t = C'_{it}(\bar{q}_{it} - q_{it}) - \mu_t, \tag{32}$$

for all $t < T$. I say that prices should roughly equal the allowances price because when $\mu_t \neq 0$, the secondary market constraint is binding and not every firm can buy or sell the number of allowances it wants, driving a wedge between the allowance price and marginal abatement costs.

Observe that cost minimization forces each firm i to choose $m_{it} \leq 0$ for all $t \geq T$; all want to sell allowances if they have some. Combined with the secondary market constraint that $\sum_i m_{it} = 0$ this gives $m_{it} = 0$, as stated in Remark 1.

[9]Without loss of generality, I multiply the shadow values μ_t for the secondary market constraint (3) and ψ_{it} for the borrowing constraint by β^t.

A.2 Proofs

PROOF OF LEMMA 2

Proof. Using (28) and (29) gives:

$$p_t + \mu_t = C'_{it}(\bar{q}_{it} - q_{it}), \tag{33}$$

implying (32). Moreover, combining (30) and (29) yields:

$$p_t + \mu_t + \psi_{it} = \beta p_{t+1} + \beta \mu_{t+1}, \tag{34}$$

so $p_{t+1} = (p_t + \mu_t + \psi_{it})/\beta - \mu_{t+1}$ and this implies (8). □

PROOF OF PROPOSITION 1

Proof. Two qualitatively distinct scenarios can occur: (i) $B^P_{\bar{T}}(p^P) = 0$ and (ii) $B^P_{\bar{T}}(p^P) > 0$.

In case (i), the equilibrium price vector when the ban on emissions is advanced from T to $\bar{T}$ is the same until period $\bar{T}$: $p^P_t = \bar{p}^P_t$ for all $t < \bar{T}$. This can be proven by contradiction. Suppose $\bar{p}^P \neq p^P$. Then either (a) $\bar{p}_t < p_t$ or (b) $\bar{p}_t > p_t$ for at least one $t < \bar{T}$ which, by Lemma 2, imply that (a) $\bar{p}_t \leq p_t$ or (b) $\bar{p}_t \geq p_t$ for all $t < \bar{T}$. But by Lemma 3, case (a) implies $B_{\bar{T}}(\bar{p}^P) < 0$ whereas case (b) implies $B_{\bar{T}}(\bar{p}^P) > 0$. Either of these violates the requirement that $\bar{p}^P$ is an equilibrium price vector when the final period on emissions is $\bar{p}$. Hence, $\bar{p}^P = p^P$. Equilibrium emissions when the final period is advanced to $\bar{T}$ are therefore equal to:

$$\sum_{t=0}^{\bar{T}} q_t(p^P, \bar{T}) = \sum_{t=0}^{\bar{T}} s_t(p^P_t).$$

When the ban is at T instead, equilibrium emissions are:

$$\sum_{t=0}^{T} q_t(p^P, T) = \sum_{t=0}^{T} s_t(p^P_t).$$

Subtracting the former from the latter gives the reduction in equilibrium emissions:

$$R^P(\bar{T}, T) = \sum_{t=0}^{T} q_t(p^P, T) - \sum_{t=0}^{\bar{T}} q_t(p^P, \bar{T}) = \sum_{t=0}^{T} s_t(p^P_t) - \sum_{t=0}^{\bar{T}} s_t(p^P_t) = S^P(\bar{T}, T \mid p^P).$$

In case (ii), firms originally hold a strictly positive bank of allowances at the start of period $\bar{T}$: $B^P_{\bar{T}}(p^P) > 0$. Equilibrium under the final period $\bar{T}$ is reached when $B_{\bar{T}}(\bar{p}^P) = 0$. By Lemma 3, this implies $p^P_t > \bar{p}^P_t$ for all $t < \bar{T}$. Equilibrium emissions when the final period is $\bar{T}$ are therefore:

$$\sum_{t=0}^{\bar{T}} q_t(\bar{p}^P, \bar{T}) = \sum_{t=0}^{\bar{T}} s_t(\bar{p}^P_t)$$

Equilibrium emissions when the final period is T are instead:

$$\sum_{t=0}^{T} q_t(p^P, T) = \sum_{t=0}^{T} s_t(p^P_t) = \sum_{t=0}^{\bar{T}} s_t(p^P_t) + \sum_{t=\bar{T}+1}^{T} s_t(p^P_t).$$

Subtracting the former from the latter, the reduction in equilibrium emissions when advancing the ban from T to $\bar{T}$ is:

$$
\begin{aligned}
R^P(\bar{T}, T) &= \sum_{t=0}^{\bar{T}-1} s_t(p_t^P) + \sum_{t=\bar{T}}^{T} s_t(p_t^P) - \sum_{t=0}^{\bar{T}-1} s_t(\bar{p}_t^P) \\
&= S^p(\bar{T}, T \mid p^P) + \sum_{t=0}^{\bar{T}-1} s_t(p_t^P) - \sum_{t=0}^{\bar{T}-1} s_t(\bar{p}_t^P) \\
&> S^p(\bar{T}, T \mid p^P),
\end{aligned}
$$

where the inequality follows from the fact that $p_t^P > \bar{p}_t^P$ for all $t < \bar{T}$ and therefore, by the mechanics of a price mechanism, $s_t(p_t^P) > s_t(\bar{p}_t^P)$ for all $t < \bar{T}$.

In conclusion, either $R^P(\bar{T}, T) = S^p(\bar{T}, T \mid p^P)$ or $R^P(\bar{T}, T) > S^p(\bar{T}, T \mid p^P)$. Since $S^p(\bar{T}, T \mid p^P) \geq 0$ by construction, this gives the result. $\square$

PROOF OF PROPOSITION 2

Proof. Two qualitatively distinct scenarios can occur: (i) $B_{\bar{T}}^Q(p^Q) = 0$ and (ii) $B_{\bar{T}}^Q(p^Q) > 0$. Because these scenarios, as well as their analyses, are similar to those discussed in the proof of Proposition 1, I will be short here.

In case (i), $B_{\bar{T}}^Q(p^Q) = 0$ and therefore $\bar{p}_t^Q = p_t^Q$ for all $t < \bar{T}$. The reduction in equilibrium emissions when the final period is $\bar{T}$, compared to when it is T, is therefore:

$$
\begin{aligned}
R^Q(\bar{T}, T) &= \sum_{t=0}^{T} q_t(p^Q, T) - \sum_{t=0}^{\bar{T}} q_t(p^Q, \bar{T}) \\
&= \sum_{t=0}^{T} s_t(B_t^Q(p^Q)) - \sum_{t=0}^{\bar{T}} s_t(B_t^Q(p^Q)) . \\
&= S^Q(\bar{T}, T \mid p^Q)
\end{aligned}
$$

In case (ii), $\bar{T}$: $B_{\bar{T}}^Q(p^Q) > 0$. Equilibrium under the final period $\bar{T}$ is reached when $B_{\bar{T}}^Q(\bar{p}^Q) = 0$. By Lemmas 4 and 2, this implies $p_t^Q > \bar{p}_t^Q$ for all $t < \bar{T}$. The reduction in equilibrium emissions when the final period is $\bar{T}$, compared to when it is T, is therefore:

$$
\begin{aligned}
R^Q(\bar{T}, T) &= \sum_{t=0}^{T} q_t(p^Q, T) - \sum_{t=0}^{\bar{T}} q_t(\bar{p}^Q, \bar{T}) \\
&= \sum_{t=0}^{T} s_t(B_t^Q(p^Q)) - \sum_{t=0}^{\bar{T}} s_t(B_t^Q(\bar{p}^Q)) . \\
&= S^Q(\bar{T}, T \mid p^Q) + \sum_{t=0}^{\bar{T}} s_t(B_t^Q(p^Q)) - \sum_{t=0}^{\bar{T}} s_t(B_t^Q(\bar{p}^Q)) \\
&< S^Q(\bar{T}, T \mid p^Q),
\end{aligned}
$$

where the inequality is a consequence of the fact that $p_t^Q > \bar{p}_t^Q$ for all $t < \bar{T}$, so $B_t^Q(p^Q) > B_t^Q(\bar{p}^Q)$ for all $t < \bar{T}$ and therefore, by the mechanism of a quantity mechanism, $s_t(B_t^Q(p^Q)) < s_t(B_t^Q(\bar{p}^Q))$ for all $t < \bar{T}$.

The proof is now complete as I have shown that either $R^Q(\bar{T}, T) = S^Q(\bar{T}, T \mid p^Q)$ or $R^Q(\bar{T}, T) < S^Q(\bar{T}, T \mid p^Q)$, implying that $R^Q(\bar{T}, T)$ is bounded from above by $S^Q(\bar{T}, T \mid p^Q)$. □

PROOF OF PROPOSITION 3

Proof. We know from Proposotion 2 that $R^Q(\bar{T}, T) \leq S^Q(\bar{T}, T \mid p^Q)$. Note, then, that condition (23) gives $S^Q(\bar{T}, T \mid p^Q) = 0$. Moreover, condition (22), combined with (23), gives $B_{\bar{T}}(p^Q) > 0$. The fact that $B_{\bar{T}}(p^Q) > 0$ implies that case (ii) in the proof of Proposotion 2 applies, so $R^Q(\bar{T}, T) < S^Q(\bar{T}, T \mid p^Q)$. I have already established that $S^Q(\bar{T}, T \mid p^Q) = 0$. Hence, $R^Q(\bar{T}, T) < 0$. □

PROOF OF PROPOSITION 4

Proof. From Proposition 1, the reduction in emissions under a price mechanism is bounded from below by $S^P(T, \bar{T} \mid p^P)$. From Proposition 2, the reduction in emissions under a quantity mechanism is bounded from above by $S^Q(T, \bar{T} \mid p^Q)$. The condition that baseline equilibrium supply paths are symmetric means that (25) is satisfied. Now, using (9) and (10), (25) implies $S^P(T, \bar{T} \mid p^P) = \sum_{\bar{T}}^{T} s_t(p_t^P) = \sum_{\bar{T}}^{T} s_t(B_t^Q(p^Q)) = S^Q(T, \bar{T} \mid p^Q)$ Hence, $R^Q(\bar{T}, T) \leq S^Q(T, \bar{T} \mid p^Q) = S^P(T, \bar{T} \mid p^P) \leq R^P(\bar{T}, T)$, implying the result. □

B Uncertainty

The model in the main text assumes perfect foresight about future abatement costs. In this Appendix, I establish that all of my major results hold true (in expectations) when the future is, partly, uncertain.

Let abatement costs for firm i in period t be given by $C_{it}(a_{it} \cdot \mid \theta_t)$ such that $C_{it}(0 \mid \theta_t) = 0$, $\frac{\partial C_{it}(a_{it}|\theta_t)}{\partial a_{it}} > 0$, $\frac{\partial^2 C_{it}(a_{it}|\theta_t)}{\partial a_{it}^2} \geq 0$, and $\frac{\partial^2 C_{it}(a_{it}|\theta_t)}{\partial a_{it} \partial \theta_t} \geq 0$, for all θ_t. Here, $\theta_t \in \Theta \subseteq \mathbb{R}$ is a parameter drawn at the start of period t from a known probability distribution G on Θ. Since neither the firms nor the policymaker know $(\theta_\tau)_{\tau > t}$ in period t, there is uncertainty about future abatement costs. Let $\mathbb{E}_t$ denote the expectations operator with respect to θ_τ for $\tau > t$, so that for any function $h(\theta_\tau)$ one has $\mathbb{E}_t[h(\theta_\tau)] := \int h(\theta_\tau) dG(\theta_\tau \mid \theta_t)$. $\mathbb{E}$, without subscript, denotes the expectations operator with respect to θ_t, $t \geq 0$, at the very start of the model before θ_0 is drawn. I assume that the distribution of θ_t, all t, is independent of the timing of the final period T.[10]

The possibility of unforeseen developments in θ_t turn the dynamic cost-minimization (1)–(6) into an expected dynamic cost minimization problem:

$$\min_{q_{it}, m_{it}} \sum_{t=0}^{T} \beta^t \mathbb{E}_t \left[C_{it}(\bar{q}_{it} - q_{it} \mid \theta_t) + p_t m_{it} \mid \theta_t \right], \tag{35}$$

[10] To be clear, the final period T affects the demand for emissions as firms want to use up their allowances by the time the final period T arrives. The assumption is that the precise timing of T does not affect the abatement cost function.

subject to (2)–(6). This gives the following Bellman equation:

$$V_{it} = \min_{q_{it}, m_{it}} C_{it}(\bar{q}_{it} - q_{it} \mid \theta_t) + p_t m_{it} + \psi_{it} B_{it} + \beta \mathbb{E}_t \left[V_{it+1}(B_{t+1}, \theta_{it+1}) \mid q_{it}, m_{it}, B_{it}, \theta_t \right], \quad (36)$$

subject to (2)–(6). The first-order conditions are:

$$\frac{\partial V_{it}}{\partial q_{it}} = -C'_{it}(\bar{q}_{it} - q_{it} \mid \theta_t) + \lambda_i - \beta \mathbb{E}_t \left[\frac{\partial V_{it+1}}{\partial B_{it+1}} \right] \geq 0; \quad q_{it} \geq 0; \quad q_{it} \frac{\partial V_{it}}{\partial q_{it}} = 0, \quad (37)$$

$$\frac{\partial V_{it}}{\partial m_{it}} = p_t - \lambda_i + \beta^t \mu_t + \beta \mathbb{E}_t \left[\frac{\partial V_{it+1}}{\partial B_{it+1}} \right] = 0, \quad (38)$$

$$\frac{\partial V_{it}}{\partial B_{it}} = \psi_{it} + \beta \mathbb{E}_t \left[\frac{\partial V_{it+1}}{\partial B_{it+1}} \right] = 0. \quad (39)$$

Combining (37) and (38) gives that marginal abatement costs should equal to allowance price, $C'_{it}(\bar{q}_{it} - q_{it} \mid \theta_t) = p_t$. Let $q_{it}(p_t, T \mid \theta_t)$ denote the solution to firm i's problem, that is, $q_{it}(p_t, T \mid \theta_t)$ is the level of emissions chosen by firm i in period t given the allowance price p_t, the final period T, and the fundamental θ_t. Given the assumptions on C_{it}, $q_{it}(p_t, T \mid \theta_t)$ satisfies:

$$\frac{\partial q_{it}(p_t, T \mid \theta_t)}{\partial p_t} < 0 \quad \text{and} \quad \frac{\partial q_{it}(p_t, T \mid \theta_t)}{\partial \theta_t} > 0. \quad (40)$$

Aggregated over all firms, emissions in period t are: $q_t(p_t, T \mid \theta_t) = \sum_i q_{it}(p_t, T \mid \theta_t)$.

Lemma 5. *For all $t \in \{0, 1, 2, ..., T-1\}$ and all $\theta_t \in \Theta$,*

$$\frac{\partial q_t(p_t, T \mid \theta_t)}{\partial p_t} < 0 \quad \textit{and} \quad \frac{\partial q_t(p_t, T \mid \theta_t)}{\partial \theta_t} > 0. \quad (41)$$

Next, prices rise approximately at the discount rate (exactly if the borrowing constraint isn't binding), at least in expectations, as (38) and (39) give:

$$\mathbb{E}_t[p_{t+1}] = \frac{p_t}{\beta} + \psi_{it}. \quad (42)$$

Differentiating $\mathbb{E}_t[p_{t+1}]$ with respect to p_t gives the following lemma.

Lemma 6. *For all $t \in \{0, 1, 2, ..., T-1\}$ and all $\theta_t \in \Theta$, expectected cost-minimizing prices co-move:*

$$\frac{\partial \mathbb{E}[p_{t+1}]}{\partial p_t} > 0. \quad (43)$$

Given Lemmas 5 and 6, which extend Lemma 1 and 2 in the main text, the expected bank of unused allowances is increasing in the allowance price.

Lemma 7. *Consider two periods t and τ such that $t, \tau \in \{1, 2, ..., T-1\}$. Lemmas 3 and 4 hold true in expectations for any given θ_t. That is, $\mathbb{E}_t \left[\frac{\partial B_t^P}{\partial p_\tau} \right] > 0$ and $\mathbb{E}_t \left[\frac{\partial B_t^Q}{\partial p_\tau} \right] > 0$.*

Proof. I will prove the result for price mechanisms. The reader can then verify that the result on quantity mechanisms extends as well.

First recall that $b_\tau = s_\tau^P(p_\tau) - q_\tau(p_\tau, T)$ and $B_t = \sum_{s=0}^{t-1} b_s$. Therefore, $\partial b_\tau / \partial p_\tau > 0$ and, using this,

$$\begin{aligned} \mathbb{E}_t \left[\frac{\partial B_t^P}{\partial p_\tau} \right] &= \sum_{s=0}^{t-1} \mathbb{E} \left[\frac{\partial s_s^P(p_s)}{\partial p_\tau} - \frac{\partial q_s(p_s, T)}{\partial p_\tau} \right] \\ &= \sum_{s=0}^{t-1} \mathbb{E} \left[\frac{\partial s_s^P(p_s)}{\partial p_s} \frac{\partial p_s}{\partial p_\tau} - \frac{\partial q_s(p_s, T)}{\partial p_s} \frac{\partial p_s}{\partial p_\tau} \right] > 0, \end{aligned} \tag{44}$$

where the inequality follows from Lemmas 6 and 7. □

B.1 Equilibrium: Price Mechanisms

Because of the uncertaintities about future abatement costs and hence demand, it is not possible to state the conditions for equilibrium in terms of a deterministic price vector. Instead, I will focus on expected emissions over time before the start of the game.

$$\sum_{t=0}^{T} \mathbb{E} \left[q_\tau(p_\tau^P, T \mid \theta_t) \mid \theta_0 \right] = \sum_{\tau=t}^{T} \mathbb{E} \left[s_t^P(p_t^P) \mid \theta_0 \right], \tag{45}$$

where $p^P = (p_t^P)$ now denotes the *expected* equilibrium price vector (in period 0). For two period t_1 and t_2 such that $t_1 < t_2$, define

$$\mathbb{E} \left[S^P(t_1, t_2 \mid p) \right] = \sum_{t=t_1}^{t_2 - 1} \mathbb{E} \left[s_t^P(p_t) \right]. \tag{46}$$

That is, $S^P(t_1, t_2 \mid p)$ is the expected supply of emissions allowances between periods t_1 and t_2 when the (expected) price vector is p.

Given an earlier final period $\bar{T} < T$ and the associated expected equilibrium price vector $\bar{p}^P$, define the expected reduction in equilibrium emissions as:

$$\mathbb{E} \left[R^P(\bar{T}, T) \right] = \sum_{t=0}^{T} \mathbb{E} \left[q_t(p_t^P, T \mid \theta_t) \right] - \sum_{t=0}^{\bar{T}} \mathbb{E} \left[q_t(\bar{p}_t^P, \bar{T} \mid \theta_t) \right]. \tag{47}$$

Proposition 5. *Consider a cap and trade scheme that operates a* price mechanism. *Consider two periods T and $\bar{T}$ such that $\bar{T} < T$. Let p^P denote the expected equilibrium allowance price vector when the final period is T. Then,*

$$\mathbb{E} \left[R^P(\bar{T}, T) \right] \geq \mathbb{E} \left[S^P(\bar{T}, T \mid p^P) \right] \geq 0. \tag{48}$$

That is, the expected reduction in equilibrium emissions in response to an earlier final period is positive and bounded from below under a price mechanism.

B.2 Equilibrium: Quantity Mechanisms

Before the start of the game, the expected equilibrium is identified by:

$$\sum_{t=0}^{T} \mathbb{E}\left[q_\tau(p_\tau^Q, T \mid \theta_t) \mid \theta_0\right] = \sum_{\tau=t}^{T} \mathbb{E}\left[s_t^Q(p_t^Q) \mid \theta_0\right], \tag{49}$$

where $p^Q = (p_t^Q)$ now denotes the *expected* equilibrium price vector (in period 0). For two period t_1 and t_2 such that $t_1 < t_2$, define

$$\mathbb{E}\left[S^Q(t_1, t_2 \mid p)\right] = \sum_{t=t_1}^{t_2-1} \mathbb{E}\left[s_t^Q(p_t)\right]. \tag{50}$$

That is, $S^Q(t_1, t_2 \mid p)$ is the expected supply of emissions allowances between periods t_1 and t_2 when the (expected) price vector is p.

Given an earlier final period $\bar{T} < T$ and the associated expected equilibrium price vector $\bar{p}^Q$, define the expected reduction in equilibrium emissions as:

$$\mathbb{E}\left[R^Q(\bar{T}, T)\right] = \sum_{t=0}^{T} \mathbb{E}\left[q_t(p_t^Q, T \mid \theta_t)\right] - \sum_{t=0}^{\bar{T}} \mathbb{E}\left[q_t(\bar{p}_t^Q, \bar{T} \mid \theta_t)\right]. \tag{51}$$

Proposition 6. *Consider a cap and trade scheme that operates a* quantity mechanism*. Consider two periods T and $\bar{T}$ such that $\bar{T} < T$. Let p^Q denote the expected equilibrium allowance price vector when the final period is T. Then,*

$$\mathbb{E}\left[R^Q(\bar{T}, T)\right] \leq \mathbb{E}\left[S^Q(\bar{T}, T \mid p^Q)\right]. \tag{52}$$

That is, the expected reduction in equilibrium emissions in response to an earlier final period is bounded from above (and possibly negative) under a quantity mechanism.

Let $\mathbb{E}[f^P(T)] = \min_t\{t : s_\tau^P(\mathbb{E}[p^P]) = 0 \quad \forall \tau \geq t\}$ denote the first period in which the equilibrium supply of allowances reaches 0 permanently in expectations.

Proposition 7. *Consider a cap and trade scheme that operates a* quantity mechanism*. Consider two periods T and $\bar{T}$. Let p^Q denote the expected equilibrium allowance price vector when the final period is T. If $\mathbb{E}[f^P(T)] \leq \bar{T}$ and $q_{\bar{T}}(p_{\bar{T}}^Q, T) > 0$, then*

$$\mathbb{E}\left[R^Q(\bar{T}, T)\right] \leq 0. \tag{53}$$

B.3 Equilibrium: Prices vs Quantities

Fix a final period T. Suppose it holds that

$$\mathbb{E}\left[s_t^P(p_t^P)\right] = \mathbb{E}\left[s_t^Q(p_t^Q)\right], \tag{54}$$

for all $t \in [0, T)$.

Proposition 8. *Given a final period T, if* (54) *is true then:*

$$\mathbb{E}\left[R^P(\bar{T}, T)\right] \geq \mathbb{E}\left[R^Q(\bar{T}, T)\right],$$

for all $\bar{T} < T$.

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