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Adjustable emissions caps and the price of pollution

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ABSTRACT

Cap and trade schemes often use a policy of adjustable allowance supply with the intention to stabilize the market for allowances. We investigate whether these policies deliver with a focus on allowance prices. Motivated by existing policies, we study schemes that rely on either the allowance price (price measures) or the surplus of unused allowances (quantity measures) to adjust supply in a dynamic cap and trade market. Compared to emissions trading under a fixed cap, we find that price measures stabilize allowance prices. Quantity measures can be destabilizing. Though phrased in the context of changing interest rates, our results warn more generally against the belief that quantity measures are a suitable instrument to promote a stable cap and trade market.

1. Introduction

Increasingly many cap and trade schemes have turned away from the textbook model of a fixed emissions cap in favor of adjustable allowance supply. One important motivation behind the move toward variable supply is that such policies are thought to stabilize otherwise erratic allowance prices (Fell et al., 2012; Holt and Shobe, 2016; Cason et al., 2022; Friesen et al., 2022). But do they? This paper investigates.

There are, in theory, infinitely many ways to design a policy of adjustable allowance supply. We restrict attention to two classes of policies that dominate the landscape. The first class is that of price measures, which increase allowance supply when the price of allowances increases. The second class contains quantity measures, which reduce the supply of allowances when the surplus of unused allowances grows. Examples abound of supply policies that fit these descriptions. Price measures are used in California's ETS (Borenstein et al., 2019) and the Regional Greenhouse Gas Initiative (Friesen et al., 2022). The European Union instead opted for a quantity measure in its EU Emissions Trading System (Perino, 2018; Gerlagh and Heijmans, 2019), as did Switzerland (ICAP, 2022), and one could argue that Korea ETS's liquidity provisions are a *de facto* quantity measure (Asian Development Bank, 2018).[1]

This paper takes to task the general proposition that price and quantity measures stabilize the market for emissions. Compared to emissions trading under a fixed cap, quantity measures may destabilize allowance prices; we identify cases under which they do. Price measures unambiguously stabilize prices. We conclude that quantity measures are not the one-size-fits-all tool to deliver stable prices they are hoped – and perhaps believed – to be. Though illustrated using a changing interest rate, our results bear on the stabilizing potential of adjustable supply policies more generally.

We consider a simple model of emissions trading. The supply of allowances is determined through a price or quantity measure. Trading delivers an allowance price that equals marginal abatement costs and grows with the interest rate. All else equal, an increase

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[1] A more extensive list of schemes that operate price and quantity measures also includes China's National ETS, Germany's National ETS, New Zealand's ETS, the Massachusetts Limits on Emissions from Electricity Generator, and Switzerland's ETS.

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in the interest rate raises future allowance prices and triggers a redistribution of abatement toward the future. To support higher emissions today, the allowance price must fall. Under a price measure, this causes a reduction in allowance supply that partly offsets the initial drop in prices. A price measure hence stabilizes the market. A quantity measure instead raises supply due to the increase in emissions, pushing prices further down. Quantity measures, it follows, are destabilizing.

At their core, our results are a fundamental property of the design of cap and trade schemes not limited to a changing interest rate. We offer a general critique. Any event or policy that *ceteris paribus* reduces the demand for emissions in the future will favor bringing emissions forward in time and, consequently, suppress current-day allowance prices. The same logic that governs price effects following a change in the interest rate therefore applies to all such events or policies.

This paper fits in with the literature on adjustable supply policies. As a means to accommodate uncertainty, Pizer (2002) and Fell et al. (2012) study the design and properties of price measures while Kollenberg and Taschini (2016, 2019) and Lintunen and Kuusela (2018) study quantity measures. Some authors also compare price and quantity measures in this context, see for example Fell (2016), Holt and Shobe (2016), Abrell and Rausch (2017), and Pizer and Prest (2020). Cason et al. (2022), using a combination of theory and market experiments, study how price floors (a kind of price measure) affect investment incentives in abatement technologies. Gerlagh et al. (2021) and Perino et al. (2022a,b) show that anticipated future emissions-reducing policies may cause an overall increase in emissions under a quantity measure. Perino et al. (2022a) show that this does not happen under a price measure. Gerlagh et al. (2020) credit the EU ETS' quantity measure for its relative success in weathering the COVID-19 pandemic. Our results also connect to quantitative simulations by Gerlagh et al. (2020, 2022) and Bruninx and Ovaere (2022) on EU ETS allowance prices.

A policymaker could fix the destabilizing effect of quantity measures through direct interventions in the market. Such a solution is imperfect at best. Direct interventions can easily undermine trust in and thus destabilize the very market they were meant to stabilize. Also, why have a stabilization mechanism in the first place if it does not by itself do as it is intended to? Indeed, rather that stick with a quantity measure and opt for damage control, policymakers could adopt a price measure and preempt the problems identified here.

2. Analysis

2.1. Model

Consider a cap and trade scheme that regulates emissions by a continuum $[0, 1]$ of firms i over the course of two periods $t = 0, 1$. Let a_{it} denote the abatement effort of firm i in period t, and assume that $a_{it} \geq 0$. We define abatement relative to an exogenously given business-as-usual (BAU) level of emissions, so $a_{it} := \bar{q}_{it} - q_{it}$ where $\bar{q}_{it}$ is the BAU level of emissions chosen by firm i in period t absent any regulation and q_{it} its actual emissions. Abatement costs are given by $C(a_{it})$ and satisfy $\frac{\partial C(a_{it})}{\partial a_{it}} > 0$ and $\frac{\partial^2 C(a_{it})}{\partial a_{it}^2} > 0$. For future reference, we denote $C_t'(q_{it}) := \frac{\partial C(\bar{q}_{it} - q_{it})}{\partial q_{it}}$. Define $q_t := \int_0^1 q_{it} di$. In each period, firms choose their emissions simultaneously. To reduce notational clutter, the abatement cost functions C_{it} are assumed to be common knowledge. If desired, one could interpret C_{it} as the *expected* abatement cost function.

The abatement obligation of firm i is determined by the number of allowances it owns. There are, in practice, various ways in which allowances are allocated to the firms, but we shall be agnostic about the precise method used. Let s_{it} allowances be supplied to firms i in period t, and $s_t := \int_0^1 s_{it} di$. Allowances, once supplied, can be traded on a secondary market against a price p_t which the firms, being small, take as given. We let l_{it} denote the number of allowances bought by firm i in period t; since every allowance bought must also be sold, we have $\int l_{it} di = 0$.[2] Allowances can also be traded over time in the sense that allowances supplied but not used in period 0 are carried over to period 1 – this is called banking (Rubin, 1996). We write b_i for the amount of banking by firm i in period 0. By construction, we have $b_i = s_{i0} + l_{i0} - q_{i0}$. Total banking in period 1 is $b := \int b_i di$. While it would be realistic to assume a borrowing constraint (e.g. $b \geq 0$), we follow the literature (*c.f.* Rubin, 1996; Pizer and Prest, 2020) and allow both banking and borrowing. This assumption plays no critical role in the derivation of our results but simplifies notation.

We come to the determination of firms' behavior shortly. At this point, we note that conditions in the market for allowances affect supply. We focus on two general classes of supply policies: price and quantity measures. We define each in turn.

Definition 1 (*Price Measures*)**.** Under a price measure, the supply of allowances in period 1 is increasing in the allowance price p_0. Letting s_1^P denote period-1 supply under a price measure, we thus have $s_1^P(p_0') > s_1^P(p_0)$ if and only if $p_0' > p_0$.

Price measures are studied by Roberts and Spence (1976), Pizer (2002), Abrell and Rausch (2017), Cason et al. (2022), and Friesen et al. (2022). Practical examples are price collars. We assume that s_1^P is differentiable.

Definition 2 (*Quantity Measures*)**.** Under a quantity measure, the supply of allowances in period 1 is decreasing in the bank of allowances b. Letting s_1^Q denote period-1 supply under a quantity measure, we thus have $s_1^Q(b') > s_1^Q(b)$ if and only if $b' < b$.

[2] This is formally a constraint on firms' cost-minimization problem. Because firms are assumed to be small relative to the market, we will for convenience of notation assume this constraint is not binding.

Quantity measures were studied among others by Kollenberg and Taschini (2016), Abrell and Rausch (2017), and Pizer and Prest (2020). Examples in practice are abatement bounds (Holt and Shobe, 2016), a market stability reserve like the EU's (Gerlagh et al., 2021), or Korea ETS' liquidity provisions. We assume that s_1^Q is differentiable. We also assume that $\partial s_1^Q(b)/\partial b > -1$ for all b. This ensures that firms cannot increase their emissions in both periods simultaneously.

Although supply is determined in the market for allowances, we assume that individual firms take the supply of allowances as given. This follows from the assumption that firms are price takers, which is common in the emissions trading literature (Fell et al., 2012; Fell, 2016; Kollenberg and Taschini, 2016, 2019).

Our definitions of price and quantity measures describe existing supply policies up to a degree of approximation. The Regional Greenhouse Gas Initiative (Friesen et al., 2022) and California's ETS (Borenstein et al., 2019) use price floors and ceilings that adjust the supply of allowances only when prices threaten to move beyond those administrative boundaries. Similarly, the European Union's quantity measure adjusts supply only when banking exceeds a non-zero lower bound, and even then only indirectly through its Market Stability Reserve (Perino, 2018; Gerlagh et al., 2021). We define price and quantity measures as given for purposes of tractability. We do believe the simplified definitions used here, though abstractions, preserve the gist of the way most emissions trading systems determine allowance supply. Claims regarding the applicability of our results to actual cap and trade schemes should be read with this proviso in mind.

We are interested in the sensitivity of allowance prices to the interest rate when allowance supply is based on price and quantity measures, respectively. We also want to compare these sensitivities to the case of fixed allowance supply. To make that comparison, let supply in period 1 under a fixed cap be denoted s_1^f.

Before we proceed, a note. Although our model is simple, it is still more complicated than strictly necessary. All we really need from the demand side of the market are (i) a demand for emissions that is decreasing in the allowance price, and (ii) a dynamic arbitrage condition that establishes a positive relationship between the interest rate and the growth rate of prices over time. Together with the supply responses for price- and quantity-based adjustment policies, these properties imply our results. It follows that a simpler market representation in the guise of Weitzman (1974), extended to two periods (and with a dynamic arbitrage condition like Hotelling's rule imposed), would also yield our core insights.

2.2. Equilibrium

Firms minimize the discounted sum of costs, subject to the policy constraints:

$$\begin{aligned} \min_{q_{it}, l_{it}} \quad & \sum_{t=0}^{1} \beta^t \left[C_{it}(\bar{q}_{it} - q_{it}) + p_t l_{it} \right] \\ \text{subject to} \quad & q_{i0} = s_{i0} + l_{i0} - b_i, \\ & q_{i1} = s_{i1} + l_{i1} + b_i, \end{aligned} \tag{1}$$

where $\beta := 1/(1+r)$ is the discount factor and r the interest rate. In (1), the firm takes s_{i1} as given by assumption. Because both banking and borrowing are allowed, the two constraints in (1) admit the same shadow price and may therefore be combined. The Lagrangian for this problem then becomes,

$$\mathcal{L}_i = \sum_{t=0}^{1} \beta^t \left[C_{it}(\bar{q}_{it} - q_{it}) + p_t l_{it} \right] - \lambda \left[s_{i0} + s_{i1} + l_{i0} + l_{i1} - q_{i0} - q_{i1} \right],$$

which has the following standard first-order conditions:

$$\begin{aligned} \frac{\partial \mathcal{L}_i}{\partial q_{i0}} &= C'_{i0}(q_{i0}) + \lambda = 0, \\ \frac{\partial \mathcal{L}_i}{\partial q_{i1}} &= \beta \cdot C'_{i1}(q_{i1}) + \lambda = 0, \\ \frac{\partial \mathcal{L}_i}{\partial l_{i0}} &= p_0 - \lambda = 0, \\ \frac{\partial \mathcal{L}_i}{\partial l_{i1}} &= \beta \cdot p_1 - \lambda = 0. \end{aligned}$$

Cost-minimization dictates that marginal abatement costs equal the allowance price in each period, $C'_{it}(q_{it}) = p_t$. Let $q_{it}(p_t)$ denote the level of q_{it} that solves this condition. We define $q_t(p_t) := \int_0^1 q_{it}(p_t)di$. We similarly define $b(p_0) := s_0 - q_0(p_0)$. Strict convexity of C_{it} implies that q_{it} is unique and decreasing in p_t, so

$$\frac{\partial q_t(p_t)}{\partial p_t} < 0. \tag{2}$$

As banking and borrowing are allowed, dynamic arbitrage conditions imply that allowance prices should rise with the interest rate,

$$p_1 = (1+r) \cdot p_0, \tag{3}$$

a condition known as Hotelling's rule. Had we not allowed both banking *and* borrowing of allowances, (3) might be violated if the borrowing constraint were binding, though price movements would still be positively correlated between periods. Hotelling's rule is commonly maintained in models of dynamic emissions trading (*c.f.* Fell et al., 2012; Fell, 2016; Kollenberg and Taschini, 2016) but can be dispensed with without invalidating our results — close inspection of the analysis reveals that $\partial[p_1/p_0]/\partial r > 0$ is the core property we use. There is evidence that allowance prices are driven by the interest rate and satisfy (some rough version of) Hotelling's rule, see for example Daskalakis et al. (2009) and Gorenflo (2013). There also is indirect evidence that interest rates drive price fluctuations in the market for allowances. Mu (2007) shows that interest rates are a driver of commodity prices, which Aatola et al. (2013) show determine allowance prices.

Equilibrium is reached when supply and demand are equal; the allowance price adjusts to bring about equilibrium. Because firms are price takers, this is the competitive market equilibrium.

When allowance supply is fixed, $s_1 = s_1^f$, the equilibrium price p_t^f is found by solving

$$q_0(p_0^f) + q_1(p_1^f) = s_0 + s_1^f. \tag{4}$$

We define the equilibrium under fixed supply only as a point of reference to compare the equilibrium under price and quantity measures with. Let p_t^* denote the equilibrium allowance price in period t when supply is governed by a price measure. Thus, p_t^* is the solution to

$$q_0\left(p_0^*\right) + q_1\left(p_1^*\right) = s_0 + s_1^P(p_0^*). \tag{5}$$

The equilibrium price when supply is determined through a quantity measure is p_t^{**}, which solves:

$$q_0\left(p_0^{**}\right) + q_1\left(p_1^{**}\right) = s_0 + s_1^Q\left(b\left(p_0^{**}\right)\right). \tag{6}$$

Through (3), conditions (4), (5), and (6) reveal that equilibrium prices depend on the interest rate. It remains unclear, so far, how exactly the supply policy in place affects this relationship. Our goal is to determine the dependence of allowance prices on the interest rate more precisely.

2.3. Comparative statics

We consider first the simple case in which allowance supply is fixed. To determine the effect of fluctuations in the interest rate on the equilibrium in this case, we totally differentiate both sides of (4) with respect to the interest rate:

$$\frac{\partial q_0(p_0^f)}{\partial p_0^f}\frac{\partial p_0^f}{\partial r} + \frac{\partial q_1(p_1^f)}{\partial p_1^f}\frac{\partial p_1^f}{\partial r} = 0.$$

Using (3), we can rewrite this as:

$$\frac{\partial q_0(p_0^f)}{\partial p_0^f}\frac{\partial p_0^f}{\partial r} + \frac{\partial q_1(p_1^f)}{\partial p_1^f}\left[(1+r)\frac{\partial p_0^f}{\partial r} + p_0^f\right] = 0.$$

Collecting terms,

$$\frac{\partial p_0^f}{\partial r} = -p_0^f \cdot \frac{\frac{\partial q_1(p_1)}{\partial p_1}}{\frac{\partial q_0(p_0)}{\partial p_0} + (1+r)\frac{\partial q_1}{\partial p_1}}. \tag{7}$$

One can perform similar analyses for cap and trade schemes where supply is determined by market conditions. For the case of a price measure, totally differentiate both sides of the equilibrium condition (5):

$$\frac{\partial q_0(p_0^*)}{\partial p_0^*}\frac{\partial p_0^*}{\partial r} + \frac{\partial q_1(p_1^*)}{\partial p_1^*}\frac{\partial p_1^*}{\partial r} = \frac{\partial s_1^P(p_0^*)}{\partial p_0^*}\frac{\partial p_0^*}{\partial r}.$$

Using (3) again, reshuffling and collecting terms, we find:

$$\frac{\partial p_0^*}{\partial r} = -p_0^* \cdot \frac{\frac{\partial q_1(p_1^*)}{\partial p_1^*}}{\frac{\partial q_0(p_0^*)}{\partial p_0^*} + (1+r)\frac{\partial q_1(p_1^*)}{\partial p_1^*} - \frac{\partial s_1^P(p_0^*)}{\partial p_0^*}}. \tag{8}$$

Finally, when supply is governed by a quantity measure we get:

$$\frac{\partial p_0^{**}}{\partial r} = -p_0^{**} \cdot \frac{\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}}}{\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}} + (1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}} - \frac{\partial s_1^Q(b(p_0^{**}))}{\partial p_0^{**}}}. \tag{9}$$

We are now in a position to state our first result on the effect of the interest rate on allowance prices.

Proposition 1. *The equilibrium allowance prices p_0^f, p_0^*, and p_0^{**} are decreasing in the interest rate r:*

$$\frac{\partial p_0^f}{\partial r} < 0, \qquad \frac{\partial p_0^*}{\partial r} < 0, \qquad \frac{\partial p_0^{**}}{\partial r} < 0. \tag{10}$$

The economist will recognize in Proposition 1 a fundamental fact of finance: the price of an emissions allowance – like that of any other asset – tends to decrease, all else equal, in the interest rate. While this result is well-established for the case of a fixed emissions cap (Rubin, 1996), we extend it to policies in which allowance supply is governed by a price or quantity mechanism.[3] How is the magnitude of this dependence affected by a policy of adjustable allowance supply? The next section investigates.

2.4. (De)stabilizing price stabilization

Although Proposition 1 is informative about the direction in which allowance prices develop when the interest rate changes, it does not say which measure yields the strongest effect. Which – if any – supply policy is better able to stabilize allowance prices in the face of a changing interest rate?

Proposition 2. *There exists $\varepsilon > 0$ such that if $|p_0^* - p_0^f| < \varepsilon$ and $|p_0^{**} - p_0^f| < \varepsilon$, then equilibrium allowance prices satisfy,*

$$\frac{1}{p_0^{**}}\frac{\partial p_0^{**}}{\partial r} < \frac{1}{p_0^f}\frac{\partial p_0^f}{\partial r} < \frac{1}{p_0^*}\frac{\partial p_0^*}{\partial r} < 0, \tag{11}$$

provided ε is sufficiently small. That is, price measures strictly decrease the sensitivity of equilibrium allowance prices to the interest rate. Quantity measures strictly increase equilibrium price sensitivity.

Observe that, for this and subsequent results, the ordering of absolute effects carries over to period $t = 1$ as for equilibrium prices p_0 and p_1 one has

$$\frac{1}{p_1}\frac{\partial p_1}{\partial r} = \frac{1}{(1+r)p_0}\left[p_0 + (1+r)\frac{\partial p_0}{\partial r}\right] = \frac{1}{p_0}\frac{\partial p_0}{\partial r} + \frac{1}{1+r}$$

and $(1+r)^{-1}$ is a constant.

Proposition 2 tells us that allowance prices when supply is determined through a quantity measure are strictly more sensitive to changes in the interest rate than are allowance prices when supply is set through a price measure. Indeed, compared to an exogenously fixed emissions cap, quantity measures exacerbate rather than stabilize price fluctuations. Contrary to their intention, quantity measures destabilize cap and trade markets. Price measures do properly stabilize allowances prices.

The ordering of price effects is a general consequence of the way supply policies are designed. An increase in the interest rate raises the price of emissions in the future relative to the price today. This stimulates the demand for emissions in period 0, which suppresses the allowance price and reduces banking. By design, a reduction in banking causes an increase in supply in period 1 under a quantity measure. The increase in supply pushes allowance prices even further down, enforcing the downward pressure on prices. A price measure in contrast responds to the reduction in allowance prices by reducing supply; the tighter emissions cap partly offsets the initial drop in allowance prices and stabilizes the market.

Formally, the Proposition speaks only to the *elasticity* of allowance prices with respect to the interest rate. One may instead be more interested in absolute effects. The condition that equilibrium prices start out sufficiently close implies the following.

Corollary 1. *For $\tilde{\varepsilon} > 0$ such that $|p_0^* - p_0^f| < \tilde{\varepsilon}$ and $|p_0^{**} - p_0^f| < \tilde{\varepsilon}$, one has*

$$\frac{\partial p_0^{**}}{\partial r} < \frac{\partial p_0^f}{\partial r} < \frac{\partial p_0^*}{\partial r} < 0, \tag{12}$$

provided $\tilde{\varepsilon}$ is sufficiently small.

Starting from similar equilibrium prices, a quantity measure destabilizes the allowance price while a price measure stabilizes prices, in both relative and absolute terms.

The result that quantity measures destabilize a cap and trade market cuts to the core of several quantitative simulations that assess the impact of various shocks on allowances prices in the EU ETS due to its Market Stability Reserve (Gerlagh et al., 2020, 2022; Bruninx and Ovaere, 2022). Our results establish that the strong price effects documented by these authors are neither a consequence of the specific shocks considered, nor a malfunctioning of the EU ETS in particular; they are a general and, arguably, undesirable side effect of determining the supply of emissions allowances through a quantity measure.

Proposition 2 describes equilibrium effects for comparable baseline equilibria. Here, comparability refers to the condition that p_0^* and p_0^{**} are close enough to p_0^f. In the prominent special case of quadratic abatement costs (*c.f.* Kollenberg and Taschini, 2016; Pizer and Prest, 2020; Gerlagh et al., 2021), comparability is redundant.

Corollary 2. *If the abatement cost functions C_{it} are quadratic, then Proposition 2 holds true for all ε.*

[3] See Daskalakis et al. (2009) and Gorenflo (2013), who document a negative relationship between interest rates and returns on emissions allowances in the EU ETS.

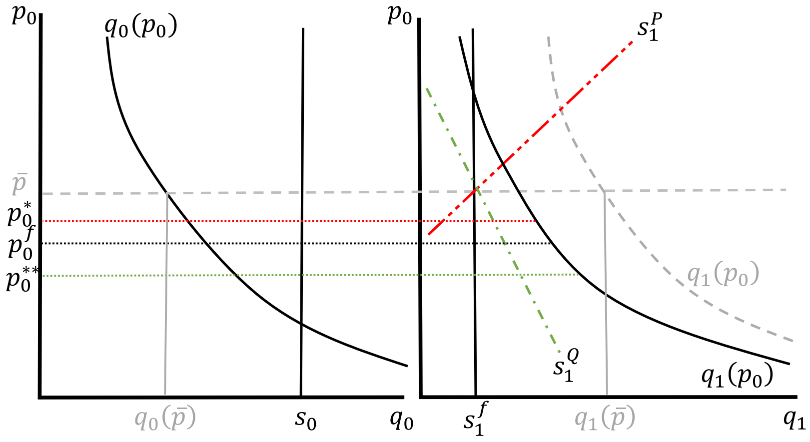

Fig. 1. Illustration of Corollary 3. The left panel plots supply and demand in period 0. The right panel plots supply and demand in period 1 as a function of $p_0 = p_1/(1+r)$. In the right panel, the downward-sloping dashed gray line gives demand as a function of p_0 given an interest rate of $\bar{r}$; the solid downward-sloping line gives demand as a function of p_0 given an interest rate of $r > \bar{r}$. The downward-sloping dashed–dotted (green) line in the right panel is supply in period 1 when determined through a quantity measure as a function of p_0. The upward-sloping dashed-double-dotted (red) line is supply in period 1 when determined through a price measure. Solid vertical lines plot exogenously fixed supply. Both panels depart from an initial equilibrium with interest rate $\bar{r}$ at which all policies deliver the same outcome; the equilibrium allowance price in this case is $\bar{p}$, emissions in period 0 are $q_0(\bar{p})$, and emissions in period 1 are $q_1(\bar{p})$. When the interest rate increases to $r > \bar{r}$, demand in period 1 as a function of p_0 shifts down (since $p_1 = (1+r)p_0$). For a given supply policy s_1^X, $X \in \{P, Q, f\}$, equilibrium is reached when $s_0 - q_0(p_0^X) = q_1(p_0^X) - s_1^X(p_0^X)$. Equilibrium prices are indicated on the vertical axis of the left panel. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The preceding results are based on there being, for every r, a sufficiently small ball around equilibrium price vectors for the various supply regimes and require continuity of the policies within said ball. Alternatively, one could assume existence of some interest rate $\bar{r}$ at which all policies deliver the same outcome. Writing equilibrium prices explicitly as a function of the interest rate, one thus has $p_0^f(\bar{r}) = p_0^*(\bar{r}) = p_0^{**}(\bar{r})$.

Corollary 3. *If there exists $\bar{r}$ such that $p_0^f(\bar{r}) = p_0^*(\bar{r}) = p_0^{**}(\bar{r})$, then*

$$\frac{\partial p_0^{**}(\bar{r})}{\partial r} < \frac{\partial p_0^{f}(\bar{r})}{\partial r} < \frac{\partial p_0^{*}(\bar{r})}{\partial r} < 0, \tag{13}$$

where the derivatives are evaluated at $r = \bar{r}$.

Corollary 3 relaxes the requirement that there be a small enough neighborhood around equilibrium prices for every r. While the focus on a particular interest rate $\bar{r}$ satisfying the imposed conditions renders the result more restrictive than Proposition 2, it buys one the freedom to relax the (implicit) assumption of continuous policies. Besides, comparing effects relative to a base case where all policies deliver the same outcome would seem most intuitive (and more in line with Weitzman (1974)). Corollary 3 is graphically illustrated in Fig. 1.

The notion that price and quantity measures can have contrasting and, sometimes, unanticipated effects was documented before. Gerlagh et al. (2021) and Perino et al. (2022a,b) describe the impact of overlapping policies on equilibrium emissions in cap and trade schemes with caps determined through a price or quantity measure. They show that anticipated future emissions policies may cause an increase in emissions under a quantity measure but not under a price measure. Because allowance prices (in period 0) are decreasing in the interest rate across supply policies, and because the demand for emissions is decreasing in the allowance price, it is not immediately obvious how the present analysis connects to these results. To establish said connection, we conclude our analysis by deriving the effect of the interest rate on emissions.

Define $e^P(p) = s_0 + s_1^P(p)$ and $e^Q(p) = s_0 + s_1^Q(s_0 - q_0(p))$ as the aggregate supply of emissions allowances when supply is governed by a price and quantity measure, respectively, when the price in period 0 is p. Given that aggregate emissions in the scheme are equal to aggregate supply in equilibrium (i.e. conditions (5) and (6)), it follows that $e^P(p_0^*)$ and $e^Q(s_0 - q_0(p_0^{**}))$ give equilibrium emissions under a price and quantity measure, respectively. Our final result relates emissions to the interest rate.

Corollary 4. *Equilibrium emissions are decreasing [increasing] in the interest rate under a price [quantity] measure:*

$$\frac{\partial e^P(p_0^*)}{\partial r} < 0 < \frac{\partial e^Q(p_0^{**})}{\partial r}. \tag{14}$$

By reducing the price of allowances in period 0 relative to period 1, an increase in the interest rate reduces the relative scarcity of allowances in period 0 without any change in fundamentals. In this sense, an increase in the interest rate is analogous to an anticipated overlapping policy that reduces the future demand for emissions without affecting supply in period 0. The effect of overlapping policies on emissions was studied, among others, by Gerlagh et al. (2021) and Perino et al. (2022a,b). In the spirit of Perino et al. (2022a), we thus find that emissions decrease under a price mechanism. Similar to Gerlagh et al. (2021) and Perino et al. (2022a), we find that emissions increase under a quantity measures. See Perino et al. (2022b) for an intuitive discussion.

3. Discussion and conclusions

Most major cap and trade schemes, and many smaller ones, have adopted measures that allow for variable allowance supply. One motivation to institute these measures is to limit allowance price variability. We investigate whether that goal is achieved by the two most prominent kind of adaptive supply policies. We find that price-based supply measures stabilize prices indeed. Quantity-based measures destabilize prices following fluctuations in the interest rate.

Quantity measures are insufficiently versatile to deal will *all* kinds of demand shocks. As a means to contain price variability resulting from events that mostly affect the instantaneous demand for emissions, quantity measures work well (Kollenberg and Taschini, 2016; Fell, 2016; Abrell and Rausch, 2017; Gerlagh et al., 2020; Pizer and Prest, 2020). In contrast, here we show that quantity measures perform poorly when the source of variation instead is an anticipated change in firms' future demand. While the EU can partially justify the claim that its quantity measure "*improves the system's resilience to major shocks by adjusting the supply of allowances to be auctioned*", it certainly does not insulate the system from *all* major shocks.[4] This finding deepens the shadow already cast over quantity measures in several recent contributions (*c.f.* Gerlagh and Heijmans, 2019; Perino et al., 2022a,b; Gerlagh et al., 2021).

Our analysis has implications for the design of cap and trade schemes beyond the context of changing interest rates. The results presented here are a general consequence of (i) the arbitrage condition, and (ii) changing future demand. In our case, the change in future (relative to current) demand was brought about through a change in the interest rate. Other events that similarly influence the relative demand for emissions over time will likewise interact with the stabilization mechanisms and bear on allowance prices and emissions.

In the wake of decades high inflation, central banks are bound to alter the course of monetary policy and increase the interest rate. Absent additional policy interventions, we predict that the associated correction in allowance prices will be substantially stronger in cap and trade schemes that rely on quantity measures to adjust supply. Perhaps it is time to reform the European Union's cap and trade market and introduce a price measure to weather the turbulent times ahead and support the EU's ambitious climate agenda.

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Appendix. Proofs

Proof of Proposition 1. We know from (2) that $\partial q_t/\partial p_t < 0$. We also know that $s_1^P(p_0)/\partial p_0 > 0$, by definition. Plugging these signs into (7) and (8) yields the result for fixed and price-measure based allowance supply.

For the case a quantity measure, (2) is still true. Hence, the final claim is correct if and only if the denominator in (9),

$$\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}} + (1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}} - \frac{\partial s_1^Q(b(p_0^{**}))}{\partial p_0^{**}},$$

is negative. Observe that we have

$$\frac{\partial s_1^Q(b(p_0^{**}))}{\partial p_0^{**}} = \frac{\partial s_1^Q(b(p_0^{**}))}{\partial b(p_0^{**})}\frac{\partial b(p_0^{**})}{\partial p_0^{**}} = -\frac{\partial s_1^Q(b(p_0^{**}))}{\partial b(p_0^{**})}\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}},$$

where the final equality follows from $b(p_0) = s_0 - q_0(p_0)$. Now recall the assumption that $\partial s_1^Q(b)/\partial b > -1$, so $1 + \partial s_1^Q(b)/\partial b > 0$ and

$$\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}} + (1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}} - \frac{\partial s_1^Q(b(p_0^{**}))}{\partial p_0^{**}} = \underbrace{\left[1 + \frac{\partial s_1^Q(b(p_0^{**}))}{\partial b(p_0^{**})}\right]}_{>0}\underbrace{\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}}}_{<0} + \underbrace{(1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}}}_{<0} < 0,$$

as we needed to show. □

Proof of Proposition 2. Define $R^*(p_0^*, p_0^f)$ as:

$$R^*(p_0^*, p_0^f) := \frac{1}{p_0^*}\frac{\partial p_0^*}{\partial r} \Big/ \frac{1}{p_0^f}\frac{\partial p_0^f}{\partial r} = \frac{\frac{\partial q_1(p_1^*)}{\partial p_1^*}}{\frac{\partial q_1(p_1^f)}{\partial p_1^f}} \frac{\frac{\partial q_0(p_0^f)}{\partial p_0^f} + (1+r)\frac{\partial q_1}{\partial p_1^f}}{\frac{\partial q_0(p_0^*)}{\partial p_0^*} + (1+r)\frac{\partial q_1(p_1^*)}{\partial p_1^*} - \frac{\partial s_1^P(p_0^*)}{\partial p_0^*}},$$

[4] Quote taken European Commission's web page.

where the expression for $R^*(p_0^*, p_0^f)$ follows from plugging in (7) and (8).

We first show that $R^*(p_0^*, p_0^f) < 1$ for all (p_0^*, p_0^f) such that $p_0^* = p_0^f$. In that case, the above simplifies to

$$R^*(p_0^*, p_0^f) = \frac{\frac{\partial q_0(p_0^*)}{\partial p_0^*} + (1+r)\frac{\partial q_1(p_1^*)}{\partial p_1^*}}{\frac{\partial q_0(p_0^*)}{\partial p_0^*} + (1+r)\frac{\partial q_1(p_1^*)}{\partial p_1^*} - \frac{\partial s_1^P(p_0^*)}{\partial p_0^*}} < 1,$$

where the inequality is immediate from the fact that $\partial s_1^P(p_0)/\partial p_0 > 0$.

Suppose then that $p_0^* \neq p_0^f$. Two cases can arise. i) $R^*(p_0^*, p_0^f) < 1$ for all (p_0^*, p_0^f), in which case the second inequality in (11) is trivially true. ii) There are (p_0^*, p_0^f) such that $R^*(p_0^*, p_0^f) \geq 1$. For any given p_0^f, let $\delta^+(p_0^f)$ denote the smallest real number such that $R^*(p_0^f + \delta(p_0^f), p_0^f) \geq 1$. Similarly, let $\delta^-(p_0^f)$ denote the smallest real number such that $R^*(p_0^f - \delta^-(p_0^f), p_0^f) \geq 1$. For all p_0^f, define $\delta(p_0^f) := \min\{\delta^+(p_0^f), \delta^-(p_0^f)\}$. Then define $\delta := \min_{p_0^f} \delta(p_0^f)$. By construction, $R^*(p_0^*, p_0^f) < 1$ for all pairs (p_0^*, p_0^f) that satisfy $|p_0^* - p_0^f| < \delta$.

Next, define $R^{**}(p_0^{**}, p_0^f)$ to be

$$R^{**}(p_0^{**}, p_0^f) := \frac{1}{p_0^{**}}\frac{\partial p_0^{**}}{\partial r} \bigg/ \frac{1}{p_0^f}\frac{\partial p_0^f}{\partial r} = \frac{\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}}}{\frac{\partial q_1(p_1^f)}{\partial p_1^f}} \frac{\frac{\partial q_0(p_0^f)}{\partial p_0^f} + (1+r)\frac{\partial q_1(p_1^f)}{\partial p_1^f}}{\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}} + (1+r)\frac{\partial q_1(p_1^{**})}{\partial p_1^{**}} - \frac{\partial s_1^Q(p_0^{**})}{\partial p_0^{**}}}.$$

If $p_0^{**} = p_0^f$, then $R^{**}(p_0^{**}, p_0^f) > 1$ as $\partial s_1^Q(b(p_0))/\partial p_0 < 0$. Suppose then that $p_0^{**} \neq p_0^f$. Here, too, two cases can arise. i) $R^{**}(p_0^{**}, p_0^f) > 1$ for all (p_0^*, p_0^f), in which case we are done. ii) There are (p_0^*, p_0^f) such that $R^{**}(p_0^{**}, p_0^f) \leq 1$. For any given p_0^f, let $\gamma^+(p_0^f)$ denote the smallest real number such that $R^{**}(p_0^f + \gamma^+(p_0^f), p_0^f) \geq 1$. Similarly, let $\gamma^-(p_0^f)$ denote the smallest real number such that $R^{**}(p_0^f - \gamma^-(p_0^f), p_0^f) \geq 1$. For all p_0^f, define $\gamma(p_0^f) := \min\{\gamma^+(p_0^f), \gamma^-(p_0^f)\}$. Then define $\gamma := \min_{p_0^f} \gamma(p_0^f)$. By construction, $R^{**}(p_0^{**}, p_0^f) > 1$ for all pairs (p_0^{**}, p_0^f) that satisfy $|p_0^{**} - p_0^f| < \gamma$. Set $\varepsilon \leq \min\{\delta, \gamma\}$. □

Proof of Corollary 4. For a price measure,

$$\frac{\partial e^P(p_0^*)}{\partial r} = \frac{\partial e^P(p_0^*)}{\partial p_0^*}\frac{\partial p_0^*}{\partial r} = \frac{\partial s^P(p_0^*)}{\partial p_0^*}\frac{\partial p_0^*}{\partial r} < 0, \tag{15}$$

where the inequality follows from combining the definition of a price measure with the results in Proposition 1.

For a quantity measure,

$$\frac{\partial e^Q(p_0^{**})}{\partial r} = \frac{\partial e^Q(s_0 - q_0(p_0^{**}))}{\partial p_0^{**}}\frac{\partial p_0^{**}}{\partial r} = -\frac{\partial s^Q(s_0 - q_0(p_0^{**}))}{\partial b(p_0^{**})}\frac{\partial q_0(p_0^{**})}{\partial p_0^{**}}\frac{\partial p_0^{**}}{\partial r} > 0, \tag{16}$$

where the inequality follows from combining the definition of a quantity measure with the results in Proposition 1 and the fact that $\partial q(p)/\partial p < 0$. □

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