# Flexible emissions caps and counterproductive policies

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#### Abstract

We prove that in any cap and trade scheme in which the emissions cap is flexible and responds to the quantity of used emissions allowances, there always exist policies complementary to the scheme that, though aimed at reducing emissions, cause an increase in emissions overall. If follows that climate policies necessarily risk being counterproductive when acting upon emissions that are already regulated through a cap and trade scheme with a quantity-based flexible emissions cap. Our result substantially generalizes and extends a number of recent findings in the literature.

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## 1 Introduction

Cap and trade is a cornerstone of modern environmental policy. By 2023, nearly 20% of global greenhouse gas emissions are regulated through a cap and trade scheme (ICAP, 2022). The predominance of emissions trading alone warrants a careful economic study of cap and trade policies. Moreover, given that no cap and trade scheme exists in a vacuum, it is equally important to understand how cap and trade schemes interact with other, complementary policies. This paper aims to develop that understanding.

In its canonical form, a cap and trade scheme issues a fixed number of emissions allowances and supplies these to firms covered by the scheme; to emit, a firm must surrender the corresponding number of allowances. The total supply of allowances thus determines the emissions cap. As emissions trading gained popularity, however, academics and policymakers became increasingly uncomfortable with the idea of a completely set and fixed emissions cap. Such rigidness, it was thought, does not reflect the fundamental notion that firms possess private information, an idea almost as old as environmental economics itself (Kwerel, 1977; Dasgupta et al., 1980). Similarly, a permanently fixed cap does not permit accommodations to unforeseen developments such as macroeconomic shocks or technological breakthroughs. For these and other reasons, a recent literature promotes the idea of flexible emissions caps.

In practice, flexible emissions caps come in two kinds: price- and quantity-based. Under a price-based emissions cap, the supply of allowances responds to the price of emissions (Roberts and Spence, 1976). Under a quantity-based cap, supply reacts to the demand for emissions.<sup>1</sup> Quantity-based emissions caps were proposed by various authors in recent years, notably including Kollenberg and Taschini (2016, 2019), Lintunen and Kuusela (2018), Heutel (2020), Pizer and Prest (2020), Karp and Traeger (2021), and Gerlagh and Heijmans. They are used in the European Union's Emissions trading System (the world's largest market for carbon in terms of value), Switzerland's ETS, and South Korea's ETS.<sup>2</sup>

The main result of this paper establishes that in any cap and trade scheme with a quantity-based flexible emissions cap, there exists an emissions-reducing policy complementary to the scheme that causes an increase in emissions overall. Hence, under a flexible emissions cap there is always the risk of implementing a counterproductive complementary climate policy.

We derive this result very generally. There is a cap and trade scheme that regulates emissions in a finite number of periods, regions, or sectors. Allowances can be traded between periods (or regions, or sectors), though we permit restrictions or frictions on trade in some ways. The supply of allowances is flexible and responds to emissions (i.e. the number of used allowances) in some deterministic way. In equilibrium, aggregate

<sup>&</sup>lt;sup>1</sup>We note that, typically, supply is increasing in demand. For purposes of generality, we shall define *any* policy in which allowance supply responds to the demand for emissions as a quantity-based emissions cap. Our results hence are not restricted to the special case of increasing supply policies.

<sup>&</sup>lt;sup>2</sup>South Korea does not formally use quantity information to update its cap, although it has historically done so according to the Asian Development Bank (2018).

emissions equal the total supply of allowances. On top of the cap and trade policy, there can be other emissions policies that, though independent of the scheme, affect the demand for emissions. Given some minor technical assumptions, this allows us to show that there always exists a vector of emissions-reducing policies that lead to an increase total in equilibrium emissions.

Our theorem substantially generalizes, and extends, several extant results on the effect of overlapping emissions policies under a quantity-based flexible cap due to Gerlagh et al. (2021), Jarke-Neuert and Perino (2020), Osorio et al. (2021), Kruse-Andersen and Jacobsen (2022), and Perino et al. (2022a). These authors study quantity-based flexible emissions caps modeled after the most prominent real-world example: the Market Stability Reserve in the EU ETS. Their results are special cases of our main result.

The remainder of the paper is structured as follows. Section 2 discusses the economic intuition for our result in a simple, two-period model. Section 3 presents our model and states our main result. Section 4 discusses and concludes. Proofs are given in the appendix.

# 2 Economic intuition

Though are analysis is technical, there is a straightforward economic intuition for our result. We provide a graphical illustration here.

Consider a dynamic cap and trade scheme that regulates emissions in two periods, 1 and 2. If the aggregate cap is fixed, say at S, the sum of emissions  $q_t$  in the two periods must equal the cap, or  $q_1 + q_2 = S$ . Assuming that trade of emissions is allowed between the two periods (i.e. both banking and borrowing are permitted), the cap is given the straight line with slope -1 in Fig 1a. An equilibrium choice of periodic emissions will always be on, not below, this line.

If instead the cap is flexible, the cap cannot be represented by a straight line with slope -1 everywhere; the cap must have a slope different from -1 at least somewhere. One possibility is given in Fig 1b, which mimics the functioning of the EU ETS.<sup>3</sup> In this figure, the line that represents the emissions cap has slope between 0 and -1, capturing the idea that a reduction in emissions in period 1 (more banking) leads to a reduction in supply in period 2, tightening the aggregate emissions cap. Let the initial equilibrium be  $E_0$ . If the demand for emissions in period 1 is reduced, for example due to a complementary policy in that period, emissions in period 1 decrease; graphically, the market moves to the left along the dashed horizontal arrow to  $A_1$ . The emissions allocation  $A_1$  cannot be an equilibrium, however, as firms are not using the total amount of allowances allocated to them. Hence, rather than reach an equilibrium at  $A_1$ , emissions prices will adjust so that total allowances remain consistent with the

<sup>&</sup>lt;sup>3</sup>The EU ETS includes a Market Stability Reserve (MSR) and a corresponding cancellation mechanism, which basically implies that if emissions in early years are sufficiently low compared to the annual supply of allowances, allowances will be canceled and hence the aggregate cap reduced. For more details, see e.g. (Perino, 2018; Gerlagh and Heijmans, 2019).



Figure 1: This figure presents allowances in two periods (or regions or sectors). Panel (a) has an exogenous cap. Panel (b) has an endogenous cap rule depicted through the solid curved. If some policy reduces demand for allowances in period 1 (region/sector) from  $E_0$  to A (arrow to left), then prices adjust endogenously and total emissions decrease in the new equilibrium  $E_1$ . Panel (c) has a general endogenous cap rule such that if, in equilibrium  $E_0$  emissions in the first period decline, aggregate emissions increase.

aggregate cap rule. In the figure, prices go down and emissions go up from  $A_1$  to  $E_1$ , the new equilibrium. Importantly, however, because the aggregate cap has slope less than -1 (in absolute value), total emissions in  $E_1$  are strictly less than in  $E_0$ : the complementary policy is effective. While prices decrease due to the emissions reducing policy, an endogenous cap dampens this price effect and ensures that aggregate emissions still decrease after the price correction.

Now imagine that the policy does not target demand in period 1 but instead aims to reduce emissions in period 2; assume that this effect is anticipated in period 1, i.e. that the policy is announced in advance of its actual implementation. Starting again from the initial equilibrium  $E_0$ , the market's initial response to the policy will be to decrease period 2 emissions to the point  $A_2$ . Since total emissions are less that the aggregate cap in  $A_2$ , this cannot be an equilibrium and prices will adjust. Once prices have adjusted, demand in both periods increases again relative to  $A_2$  and the market reaches a new equilibrium in  $E_2$ . Importantly, aggregate emissions in  $E_2$  are strictly higher than in  $E_0$  (because the cap has a slope > -1). Hence, the climate policy is counterproductive: though aimed at reducing emissions, in equilibrium the policy causes an *increase* in total emissions.

Conceptually, if the aggregate cap is not constant there always exists an inward moving arrow (i.e. policy) that, once prices adjust, triggers an outward shift of the equilibrium. This paper formalizes that geometric intuition in a general setting with multiple linked periods, regions or sectors.

## 3 Analysis

Consider a cap and trade scheme that regulates emissions in a finite number of periods, regions, or sectors. We will use index i = 1, 2, ... and for convenience refer to a specific i as a "period". The index is general, i can also cover multiple periods, multiple regions, and multiple sectors, jointly. We assume that prices are positively co-moving between periods, i.e. that  $\partial p_{i+1}/\partial p_i > 0.4$  The positive co-movement of prices is a consequence of trading between two periods i and j,  $i \neq j$ , henceforth assumed. We can thus conveniently capture all price information by a single scalar variable p. The equilibrium, to be defined shortly, is then uniquely identified by a scalar  $p^*$ .

Let  $d(p, \lambda)$  denote the demand vector for emissions allowances, which depends upon the price p and the vector of emissions policies  $\lambda = (\lambda_i)_i$ . We normalize  $\lambda$ so that  $\partial d/\partial \lambda = u$ , where  $u = (1, 1, ..., 1)^T$  is the transposed all-ones-vector with 1 everywhere.<sup>5</sup> We assume that the vector  $\lambda$  is known in all periods i; in the temporal interpretation of our model, we hence assume that policies are anticipated. We also assume that the demand for allowances decreases in prices,  $d' \equiv \partial d/\partial p < 0$ . Aggregate demand is denoted  $D(p, \lambda) = u^T d$ .

We allow for the number of periods with strictly positive demand for emissions to be endogenous, say T. We assume that demand has a (period-specific) finite choke price  $\bar{p}_i$ . We abstract from negative emission technologies, so emissions in every period are at least 0.

Observe that the demand for emissions allowances is a function of allowance prices and other policies. Further, the actual amount of emissions eventually emitted is determined by the supply of emissions allowances, and the price adjusts to equate supply and demand. Let  $s_i$  denote the supply of allowances in period i, and  $\mathbf{s} = (s_i)_i$ the vector of supply in all periods. We call the total supply of allowances 'the emissions cap', given by  $\mathbf{u}^T \mathbf{s}$ .

We are interested in flexible emissions caps, i.e., cap and trade schemes in which the supply of allowances s depends on the demand for allowances d. Under a quantity based flexible emissions cap, per period (region, sector) per-period allowances supply s depends on the vector of per-period emissions: s = s(d). At the aggregate level, aggregate allowances supply S depends on surrendered allowances d:

$$S = S(\boldsymbol{d}) \equiv \boldsymbol{u}^T \boldsymbol{s}(\boldsymbol{d}). \tag{1}$$

When i denotes time, supply may depend on current and past emissions; importantly, this dependence is not restricted to the more aggregated "banking" of allowances

<sup>&</sup>lt;sup>4</sup>A special case of this assumption for dynamic emissions trading is the canonical version of Hotelling's rule which says that  $p_{i+1} = (1 + r)p_i$ , with r the interest rate. Hotelling's rule is maintained in the analyses of Gerlagh et al. (2021), Osorio et al. (2021), and Perino et al. (2022a,b). For multi-sector or multi-region cap and trade schemes, the special case usually maintained is  $p_{i+1} = p_i$  for all *i*, see e.g. Perino et al. (2022a).

<sup>&</sup>lt;sup>5</sup>Such normalization is possible if demand-reducing policies can be described through a set of independent policies that spans the emissions space.

(Gerlagh et al., 2021; Perino et al., 2022a). When i denotes regions or sectors, supply in region i may depend on emissions in any region j. When i denotes a matrix over regions and time, allowances supply may depend on current and past emissions in both the own and other regions. Our notation is general and does not discriminate between these cases.

The market is in equilibrium when excess demand, D-S, equals zero. It is assumed that polluters are sufficiently small to take prices as given. The price of allowances adjusts to bring about equilibrium.

**Definition 1** (Equilibrium). The equilibrium price  $p^*$  solves

$$D(p^*, \boldsymbol{\lambda}) = \boldsymbol{u}^T \boldsymbol{d}(p^*, \boldsymbol{\lambda}) = \boldsymbol{u}^T \boldsymbol{s}(\boldsymbol{d}(p^*, \boldsymbol{\lambda})) = S(\boldsymbol{d}(p^*, \boldsymbol{\lambda})).$$
(2)

The rate of adjustment of the emissions cap with respect to emissions is given by the gradient  $S' \equiv \boldsymbol{u}^T \nabla \boldsymbol{s}(\boldsymbol{d})$ , with  $(\nabla \boldsymbol{s}(\boldsymbol{d}))_j^i = \partial s_i / \partial d_j$ ,  $S'_j = \sum_i \partial s_i / \partial d_j$ , and we adopt the notational convention that  $\boldsymbol{x}_i$  (subscript *i*) denotes the *i*<sup>th</sup> element of a vector  $\boldsymbol{x}$ , so a scalar, whereas  $\boldsymbol{x}^i$  (superscript *i*) denotes th *i*<sup>th</sup> column vector of a matrix  $\boldsymbol{x}$ . Hence,  $\boldsymbol{x}_j^i$  the *j*<sup>th</sup> element of the *i*<sup>th</sup> vector of  $\boldsymbol{x}$ .

The emissions cap is fixed, or exogenous, if the supply gradient is either 0, S' = 0, or more generally proportional (but not equal) to the all-ones-vector,  $S' = \beta \boldsymbol{u}$  with  $\beta \neq 1$ . The latter is not obvious but is easily explained. From  $S' = \beta \boldsymbol{u}$  it follows that  $S = S_0 + \beta \boldsymbol{u}^T \boldsymbol{d}$  so that  $D \equiv \boldsymbol{u}^T \boldsymbol{d} = S_0/(1-\beta)$  (since D = S in equilibrium). For  $\beta = 1$ the equilibrium does not exist iff  $S_0 \neq 0$  and is indeterminate iff  $S_0 = 0$ .

**Definition 2** (Flexible emissions cap). A quantity-based emissions cap is flexible if and only if the supply gradient is not proportional to the unit vector, i.e.  $S' \neq \beta u$  almost everywhere.

A natural approach would be to study properties of the equilibrium through the response of the equilibrium condition with respect to prices  $p^*$ . However, as the flexible emissions cap depends on the demand vector  $\boldsymbol{d}$  directly, the present analysis is better served by considering the response of the equilibrium condition with respect to demand  $\boldsymbol{d}$  itself. To this end, note that by definition, the equilibrium is characterized by the condition  $\boldsymbol{u}^T \boldsymbol{d}(p^*, \boldsymbol{\lambda}) - S(\boldsymbol{d}(p^*, \boldsymbol{\lambda})) = 0$ . The gradient of the equilibrium in demand space is hence  $\boldsymbol{u} - S'$ .

From the equilibrium definition, a change in demand  $\Delta \boldsymbol{d}$  is consistent with equilibrium if and only if  $(\boldsymbol{u} - S')^T \Delta \boldsymbol{d} = 0$ . Let  $\Delta \boldsymbol{d} \ge 0$  denote the event where changes in demand are non-negative in all periods *i* and strictly positive in at least one *i*. If there exists a  $\Delta \boldsymbol{d} \ge 0$  that is consistent with the equilibrium changes in emissions, i.e. that satisfies  $(\boldsymbol{u} - S')^T \Delta \boldsymbol{d} = 0$ , then this would imply a "free lunch" for the polluters whose emissions are covered by the scheme. We rule out such free lunches. That is, we make sure there does not exist a  $\Delta \boldsymbol{d} \ge 0$  such that  $(\boldsymbol{u} - S')^T \Delta \boldsymbol{d} = 0$ .

Assumption 1 (No free lunch). Supply of allowances does not increase one-to-one or more with actual emissions: S' < u.

We are interested in the effects of policy-induced changes in demand. Recall that  $\boldsymbol{\lambda} = (\lambda_i)_i$  denotes the vector of emissions policies. Let  $\alpha_i$  denote the response of the equilibrium price  $p^*$  to a change in  $\lambda_i$ ,

$$\boldsymbol{\alpha}_i \equiv dp^*/d\boldsymbol{\lambda}_i. \tag{3}$$

Similarly, let  $\gamma^i$  denote the change in the vector of equilibrium emissions in response to a change in  $\lambda_i$ ,

$$\boldsymbol{\gamma}^i \equiv d\boldsymbol{d}^*/d\boldsymbol{\lambda}_i,\tag{4}$$

where  $\boldsymbol{\gamma}_{j}^{i}$  denotes the change in demand in period j from a policy-induced demand change in period i.

We let  $\Gamma$  denote the matrix of all policy-induced changes in emissions so that, with a slight abuse of notation, we may write  $\Gamma e^i \equiv \gamma^i$ , where  $e^i$  is the unit vector with zeros everywhere but 1 at the  $i^{th}$  place (that is,  $e^i$  is the  $i^{th}$  column of the *T*-dimensional identity matrix).

Upon differentiating the equilibrium condition (2) with respect to  $\lambda_i$ , one obtains a useful lemma.

**Lemma 1.** All policy-induced changes in demand are orthogonal to the demand space gradient, or

$$(\boldsymbol{u} - S')^T \boldsymbol{\gamma}^i = 0, \tag{5}$$

for all i.

Because Lemma 1 holds true for all i, an immediate implication is that any linear combination of the set  $\{\gamma^i\}_i$  also satisfies the orthogonality property. We thus have

$$(\boldsymbol{u} - S')^T \Gamma = \boldsymbol{0}.$$
 (6)

Lemma 2 shows that equilibrium prices and demand respond intuitively to policy changes.

Lemma 2. Prices increase with demand-increasing policies,

$$\boldsymbol{\alpha} = -\frac{(\boldsymbol{u} - S')^T}{(\boldsymbol{u} - S')^T \boldsymbol{d}'} > 0, \tag{7}$$

and own-period demand increases, while other-period demand decreases:

$$\boldsymbol{\gamma}_{j}^{i} < 0 \quad for \quad j \neq i,$$
(8)

$$\boldsymbol{\gamma}_i^i > 0. \tag{9}$$

Lemma 2 is proved in the Appendix.

A green paradox arises when an emissions-reducing policy in some period leads to an increase in aggregate equilibrium emissions. Such a policy is counterproductive relative to its original intention. **Definition 3** (Green paradox). There is a green paradox if a demand-decreasing policy,  $d\lambda < 0$ , causes an increase in aggregate emissions,  $dD = d(\mathbf{u}^T \mathbf{d}^*) > 0$ .

The main result of the paper is that for any cap and trade scheme with a flexible emissions cap based on quantities, one can find a demand-reducing policy that induces a green paradox.

**Theorem 1.** In every cap and trade system with a quantity-based flexible emissions cap and without a free lunch, there exists a policy  $d\lambda < 0$  that induces a green paradox,  $d(\mathbf{u}^T \mathbf{d}^*) > 0$ .

We prove our Theorem in the Appendix. Its main implication of is straightforward: any climate policy aimed at reducing emissions risks being counterproductive when targeted at emissions that are already regulated through a cap and trade scheme with a flexible and quantity-based emissions cap.

# 4 Discussion and Conclusions

We prove that in *any* cap and trade scheme in which the emissions cap is flexible and responds to quantities, there exist policies that, though aimed at reducing emissions, cause an increase in emissions overall. Thus, climate policies always risk being counterproductive when they interact with a quantity-based flexible emissions cap. Our main result establishes fundamental limitations on the effectiveness of using quantity information determine emissions targets in cap and trade schemes. In the context of the EU ETS, special cases of our result were previously found in Gerlagh et al. (2021), Osorio et al. (2021), and Perino et al. (2022a,b).

Because of both asymmetric information between the planner and regulated firms as well as uncertainty about future abatement costs, flexible emissions caps intuitively sound like a good idea. Why might things still go awry? In our view, the fundamental problem that underlies Theorem 1 can be found in information economics. Economists motivate flexible emissions cap as a means through which the policymaker can incorporate polluting firms' private information in the setting of an emissions cap. The idea is that private knowledge about, for example, abatement costs can be inferred from market outcomes. Observing these outcomes, the policymaker might therefore make the policy more efficient by better aligning the cap on emissions with perceived market fundamentals. This is the core of the argument underlying proposals for flexible emissions caps, see for example Kollenberg and Taschini (2016, 2019), Lintunen and Kuusela (2018), Gerlagh and Heijmans, Pizer and Prest (2020), and Karp and Traeger (2021).

This simple logic turns out to fail: the demand for emissions is an ambiguous signal of market fundamentals at best. The use of emissions allowances is not informative about the aggregate demand for emissions but only about relative demand, that is, the development of the demand for emissions over time (or between regions and sectors). An increase in emissions today does not necessarily signal an increase in the demand for emissions overall; it merely indicates that the demand for emissions today, relative to the future, has increased. The informational ambiguity of emissions similarly explains the result due to Heijmans (2023a) that a quantity-based emissions cap, though intended to stabilize allowance prices, in effect destabilizes the market for emissions.

We note that our result is not necessarily critical of flexible emissions caps per se. The analysis presented here exclusively deals with quantity-based flexibility of the emissions cap. Various existing schemes, including the Regional Greenhouse Gas Initiative, California's ETS, and the ETS in Quebec, rely on allowance prices to update the cap on emissions. This paper does not study price-based emissions caps, but we expect that our result does not generalize to flexibility based on prices. We ground our expectation in a variety of recent results on the relative merits of priceand quantity-based flexible emissions caps. Perino et al. (2022a) find that a pricebased flexibility mechanism does not lead to an increase in emissions in response to complementary climate policies. Heijmans (2023a) shows that price-based emissions caps stabilize allowance prices whereas quantity-based caps do the opposite. Heijmans (2023b) establishes that price-based emissions caps interact intuitively with the duration of a cap and trade scheme; quantity-based caps do not.

The key policy takeaway of this paper is that these cap and trade schemes can interact with complementary climate policies in non-intentional directions, possibly increasing emissions when a reduction is aimed for. This warrants care when designing cap and trade schemes or introducing complementary climate policies in jurisdictions that already rely on emissions trading.

# A Proofs

### PROOF OF LEMMA 2

*Proof.* First differentiate both sides of the equilibrium condition (2). This gives

$$-(\boldsymbol{u}-S')^T\boldsymbol{d}'\boldsymbol{\alpha}_i = (\boldsymbol{u}-S')^T\boldsymbol{\alpha}_i, \qquad (10)$$

and we note that  $-(\boldsymbol{u} - S')^T \boldsymbol{d}' > 0$  by  $\boldsymbol{d}' < 0$  and Assumption 1; (7) follows.

From (7), (8) immediately follows as d' < 0 and  $d\lambda_j = 0$  for  $j \neq i$ . Combined with Assumption 1, this implies (9).

### PROOF OF THE THEOREM

*Proof.* We prove the result by contradiction. Assuming there is no green paradox, we will show that, if so, one can construct a demand-reducing policy  $d\lambda < 0$  that decreases emissions in all periods  $d\mathbf{d} < 0$ . Since this is a direct contradiction of Assumption 1, that will conclude the proof.

We first observe that if there is no green paradox, then all policies that reduce demand in some period *i* decrease aggregate emissions. In this case, the matrix  $\Gamma$  is diagonally dominant over its columns and  $\boldsymbol{u}^T \boldsymbol{\gamma}^i \geq 0$  for all *i*.

We next define normalized policies and equilibrium responses. Let  $\boldsymbol{\kappa}^i \equiv d\boldsymbol{\lambda}^i/\boldsymbol{\gamma}_i^i < 0$ and  $\boldsymbol{\eta}^i \equiv \boldsymbol{\gamma}^i/\boldsymbol{\gamma}_i^i$  so that if  $\boldsymbol{\kappa}^i = -\boldsymbol{e}^i$ , the policy reduces demand by one unit in period *i*. Let *H* be the matrix of normalized responses,  $(H\boldsymbol{e}^i)_j = \boldsymbol{\eta}_j^i$ . The matrix *H* is also diagonally dominant over its columns, a property it inherits from  $\Gamma$ , with ones on the diagonal and negative numbers everywhere else. In this notation, the effect of a policy vector  $d\boldsymbol{\lambda} < 0$  on equilibrium emissions can be described through  $d\boldsymbol{d} = H\boldsymbol{\kappa}$ .

Choose the natural number A such that any policy which directly reduces demand in period *i* by one unit will reduce aggregate demand by at least A units, or  $A = \min_i \{ \mathbf{u}^T H \mathbf{e}^i \}$ . Thus, A is the lower bound for the cumulative effectiveness of a policy in any period *i*. Because we assume that there is no green paradox, we have A > 0.

Recursively construct a series of vectors  $\boldsymbol{z}^k$ , with  $k = 1, ..., \infty$ , so that the series converges to  $\boldsymbol{z}^k \to \boldsymbol{\kappa} < 0$ , and  $H\boldsymbol{\kappa} < 0$ . We start for k = 1 with  $\boldsymbol{z}^1 = -\boldsymbol{e}^1$ . That is, the policy  $\boldsymbol{z}^1$  decreases demand in the first period by one unit and increases demand in all other periods, but aggregate demand is decreased  $\boldsymbol{u}^T H \boldsymbol{z}^1 < -A < 0$ . This immediately implies that  $\sum_i \max\{0, (H\boldsymbol{z}^1)_i\} < (1 - A)$ , i.e. the sum of all positive elements of H is bounded from above.

Assume that in step k, we know that (i)  $\boldsymbol{u}^T H \boldsymbol{z}^k < 0$ , and (ii) the sum of all positive elements is bound from above by  $\sum_i \max\{0, (H\boldsymbol{z}^k)_i\} < (1-A)^k$ . Given these two conditions, once may construct the next ((k+1)-th) element of the sequence in such a way that the properties (i)  $\boldsymbol{u}^T H \boldsymbol{z}^{k+1} < 0$  and (ii)  $\sum_i \max\{0, (H\boldsymbol{z}^{k+1})_i\} < (1-A)^{k+1}$  are transferred to the next inductive step. To see this, consider all positive elements of  $\boldsymbol{u}^T H \boldsymbol{z}^k$ , that we want to neutralize. Thus, let  $\boldsymbol{z}^{k+1}$  be defined by  $(\boldsymbol{z}^{k+1} - \boldsymbol{z}^k)_i = -\max\{0, (H\boldsymbol{z}^k)_i\} < 0$ . The required properties follow immediately from this construction:

$$\boldsymbol{u}^{T}H\boldsymbol{z}^{k+1} = \boldsymbol{u}^{T}H\boldsymbol{z}^{k} + \boldsymbol{u}^{T}H(\boldsymbol{z}^{k+1} - \boldsymbol{z}^{k}) < 0$$
(11)

$$\sum_{i} \max\{0, (H\boldsymbol{z}^{k+1})_i\} < (1-A)\sum_{i} \max\{0, (H\boldsymbol{z}^k)_i\} < (1-A)^{k+1}$$
(12)

Finally, we must show that the limit  $\kappa$  of  $z^k$  is in fact well-defined. This is easy. Note that, by construction, the sequence

$$\boldsymbol{u}^{T}(\boldsymbol{z}^{k+1} - \boldsymbol{z}^{k}) = \sum_{i} \max\{0, (H\boldsymbol{z}^{k})_{i}\} < (1 - A)^{k}$$
(13)

is a Cauchy sequence. To establish convergence it hence suffices to show that the sequence is defined on a compact set. To this end, recall that in any step  $\kappa$ , the sum of all positive elements of H was bound from above by  $(1 - A)^{\kappa}$ . The aggregate increase in emissions is therefore never larger than  $\sum_{\kappa=1}^{\infty} (1 - A)^{\kappa} = 1/A < \infty$ , where the last inequality follows from the assumption that A > 0. But this means the series of vectors

 $z^k$  is defined on a closed and bounded set. By the Heine-Borel Theorem, a closed and bounded set is compact.

Having established convergence, we thus know that  $\boldsymbol{z}^k \to \boldsymbol{\kappa} < 0$  and  $H\boldsymbol{\kappa} < 0$ . In non-normalized notation, we have therefore constructed a strict demand-reducing policy  $d\boldsymbol{\lambda} < 0$  that implies a negative emissions response in all periods,  $\boldsymbol{\gamma} = \Gamma d\boldsymbol{\lambda} < 0$ . Combined with Assumption 1,  $(\boldsymbol{u} - S')^T \neq 0$ , this implies  $(\boldsymbol{u} - S')^T \boldsymbol{\gamma} < 0$ , which contradicts (5).

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