Flexible emissions caps and counterproductive climate policies

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Abstract

We give a substantial generalization of a result due to Gerlagh et al. (2021). Any cap and trade scheme in which the aggregate cap responds to the use of allowances necessarily suffers from a Green Paradox: if information on quantities is used to determine the effective cap on emission, there always exists an emissions-reducing policy complementary to the scheme that increases emissions overall.
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1 Introduction

Emissions trading is a corenerstone environmental policy across the globe. By 2022, nearly 20% of global greenhouse gas emissions are regulated through a cap and trade scheme, and roughly a third of the world's population lives under an emissions cap (ICAP, 2022). This predominance alone illustrates the importance of studying cap and trade policies through the lense of economics. Moreover, though scientists have come to understand the relative merits of cap and trade versus policies rather well, recent developments in the realms of politics and science alike have led to the proposal – and implementation – of adjustments to the classical cap and trade paradigm. These developments further uncerscore the importance of a careful investigation of cap and trade policies, as major mitigation ambitions have come to depend on the functioning of now essentially experimental markets. This paper studies a particular branch of

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policy developments – flexible emissions caps – and establishes a generally negative result regarding their effectiveness in a multi-policy landscape.

In its most basic form, a cap and trade scheme issues a fixed number of emissions allowances and distributes these to firms covered by the scheme. If a firm wants to emit, it must surrender the corresponding number of allowances. The total number of allowances supplied determines the emissions cap; by setting the cap below business as usual emissions, the policy reduces emissions.

As cap and trade stimulates firms to use their private information, the resulting market signals provide indicators of firms' private information (Kwerel, 1977; Dasgupta et al., 1980). If the cap is set to strike an (approximate) balance between the marginal benefits and costs of emissions, this information is relevant to pin down the total amount of emissions an efficient policy should allow for. Thus, a recent literature promotes the idea of flexible emissions caps. That is, of cap adjustments in response to firms' emissions decisions (Abrell and Rausch, 2017; Kollenberg and Taschini, 2016, 2019; Lintunen and Kuusela, 2018; Pizer and Prest, 2020; Heutel, 2020; Gerlagh and Heijmans, 2020).

Though a policy of cap adjustments may concord well with economic intuition, the devil is in the details. As was first shown by Gerlagh et al. (2021), cap adjustments based on quantities – that is, on the number of used/unused emissions allowances – may be counterproductive. In particular, Gerlagh et al. show that anticipated overlapping policies, aimed a reducing emissions in the future, can lead to an overall increase in emissions. Comparable result were subsequently found by Jarke-Neuert and Perino (2020), Perino et al. (2022a), and Heijmans (2022).

Our main contribution in this paper is to offer a substantial generalization of the result due to Gerlagh et al. (2021). We show that any cap and trade scheme in which the cap on emissions responds to the use of allowances necessarily suffers from a green paradox. If information on quantities is used to adjust the cap, there always exists an emissions-reducing policy complementary to the scheme that increases emissions overall.

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To establish our result in full generality, we present a technical analysis. Such a presentation notwithstanding, there is a solid economic intuition for our green paradox. Consider the simple example of a two-period dynamic cap and trade scheme with a flexible emissions cap. This example is illustrated in the figure below. When the amount of unused allowances in period 1 goes down, from the equilibrium we move to the left along the solid curve.

Demand and supply of new allowances in period (region) 2 is increased so much that total emissions go up.[1] This paper establishes this feature of an endogenous cap in a general setting with multiple linked periods, regions or sectors. It thus provides reason for caution when linking cap-and trade over space, or over time. Adding flexibility can

[1]Those familiar with European climate policy will recognize a similarity to the Emissions Trading Scheme (EU ETS) in these mechanics (Perino, 2018; Gerlagh and Heijmans, 2019).

greatly improve efficiency (Karp and Traeger, 2021), but it is essential to use price information for adjusting the cap. Flexibility based on quantity information only will always lead to a green paradox.

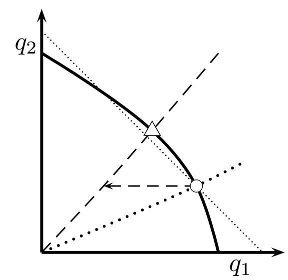

Figure 1: This figure presents allowances in two periods, regions or sectors, with an endogenous cap depicted through the solid curved line. The dotted line presents allocations with constant aggregate allowances; the circle represents an equilibrium. If demand for allowances in period (region/sector) 1 decreases (arrow to left), total emissions increase when prices adjust in the new equilibrium (triangle).

2 Analysis

Consider a cap and trade scheme that regulates emissions in a finite number of periods, regions, or sectors. We will use index $i = 1, 2, ...$ and for convenience refer to a specific i as a "period". We assume that prices are positively co-moving between periods, i.e. that $\partial p_{i+1}/\partial p_i > 0$.[2] The positive co-movement of prices is a consequence of a reasonable degree of trade in allowances between two periods i and j, $i \neq j$, being allowed; it is permissible that trade be restricted in some directions, but not entirely. This kind of free trade is henceforth assumed. We can thus conveniently capture all price information by a single scalar variable p. The equilibrium, to be defined shortly, is then uniquely identified by a scalar p^*.

Let $\boldsymbol{d}(p, \boldsymbol{\lambda})$ denote the demand vector for emissions, which depends upon the price p and the vector of emissions policies $\boldsymbol{\lambda} = (\lambda_i)_i$. We normalize $\boldsymbol{\lambda}$ so that $\partial \boldsymbol{d}/\partial \boldsymbol{\lambda} = \boldsymbol{u}$, where $\boldsymbol{u} = (1, 1, ..., 1)^T$ is the transposed all-ones-vector with 1 everywhere.[3] We assume that the vector $\boldsymbol{\lambda}$ is known in all periods i; in the temporal interpretation of our model, we hence assume that policies are anticipated. We also assume that the demand for allowances decreases in prices, $\boldsymbol{d}' \equiv \partial \boldsymbol{d}/\partial p < 0$. Aggregate demand is denoted $D(p, \boldsymbol{\lambda}) = \boldsymbol{u}^T \boldsymbol{d}$.

We allow for the number of periods with strictly positive demand for emissions to be endogenous, say T. We assume that demand has a (period-specific) finite choke

[2]For example, the canonical version of Hotelling's rule would fix $\partial p_{i+1}/\partial p_i$ at $1 + r$, with r the interest rate.

[3]That is, the set of (linearly) independent policies is equal to the number of periods.

price $\bar{p}_i$. We abstract from negative emission technologies, so emissions in every period are at least 0.

Observe that the demand for emissions is a function of allowance prices and other policies. The actual amount of emissions eventually emitted is determined by the supply of emissions allowances (and the price adjusts to equate supply and demand). Let s_i denote the supply of allowances in period i, and $\boldsymbol{s} = (s_i)_i$ the vector of supply in all periods. Note that the degree of freedom of trade between periods i and j imposed upon the market implies that in any given period emissions d_i need not equate the supply of emissions allowances s_i in that period. It follows that the total number of allowances supplied is the core constraint on emissions. We call the total supply of allowances the emissions cap, given by $\boldsymbol{u}^T\boldsymbol{s}$.

We are interested in flexible emissions caps, i.e. cap and trade schemes in which the supply of allowances $\boldsymbol{s}$ depends on the demand for allowances $\boldsymbol{d}$.

Definition 1 (Flexible emissions cap)**.** *Under a flexible emissions cap, the aggregate supply S depends on the demand for emissions $\boldsymbol{d}$:*

$$S(\boldsymbol{d}(p^*, \boldsymbol{\lambda})) = \boldsymbol{u}^T\boldsymbol{s}(\boldsymbol{d}(p, \boldsymbol{\lambda})). \tag{1}$$

The rate of adjustment of the emissions cap with respect to emissions is given by the gradient $\boldsymbol{s'} \equiv \nabla\boldsymbol{u}^T\boldsymbol{s}(\boldsymbol{d})$. The emissions cap is fixed, or exogenous, if the supply gradient is either 0, $\boldsymbol{s'} = 0$, or more generally proportional (but not equal) to the all-ones-vector, $\boldsymbol{s'} \propto \boldsymbol{u}$. The latter definition follows from the fact that for a given gradient $\boldsymbol{s'} \propto \boldsymbol{u}$, aggregate emissions must be constant under the cap.[4] A cap is flexible if it is not fixed.

The market is in equilibrium when excess demand, $D - S$, equals zero. It is assumed that polluters are sufficiently small to take prices as given. The price of allowances adjusts to bring about equilibrium.

Definition 2 (Equilibrium)**.** *The equilibrium price p^* solves*

$$D(p^*, \boldsymbol{\lambda}) = \boldsymbol{u}^T\boldsymbol{d}(p^*, \boldsymbol{\lambda}) = \boldsymbol{u}^T\boldsymbol{s}(\boldsymbol{d}(p^*, \boldsymbol{\lambda})) = S(\boldsymbol{d}(p^*, \boldsymbol{\lambda})). \tag{2}$$

It follows that if the cap is fixed, then equilibrium emissions $D(p^*, \boldsymbol{\lambda})$ are also fixed.

A natural approach would be to study properties of the equilibrium through the response of the equilibrium condition with respect to prices p^*. However, as the flexible emissions cap depends on the demand vector $\boldsymbol{d}$ directly, the present analysis is better served by considering the response of the equilibrium condition (a demand vector) with respect to demand $\boldsymbol{d}$ itself. To this end, note that by definition, the equilibrium is characterized by the condition $\boldsymbol{u}^T\boldsymbol{d}(p^*, \boldsymbol{\lambda}) - \boldsymbol{u}^T\boldsymbol{s}(\boldsymbol{d}(p^*, \boldsymbol{\lambda})) = 0$. The gradient of the equilibrium in demand space is hence $\boldsymbol{u}^T - \boldsymbol{s}'$.

From the equilibrium definition, a change in demand $\Delta\boldsymbol{d}$ is consistent with equilibrium if and only if $(\boldsymbol{u} - \boldsymbol{s}')^T\Delta\boldsymbol{d} = 0$. Let $\Delta\boldsymbol{d} \geq 0$ denote the event that changes in

[4]To see this, consider the case $S = S_0 + \beta\boldsymbol{u}^T\boldsymbol{d}$, so that $\boldsymbol{s'} = \beta\boldsymbol{u}$. This is equivalent to fixed aggregate emissions $D(p, \boldsymbol{\lambda}) = S_0/(1 - \beta)$.

demand are non-negative in all periods i and strictly positive in at least one i. If there exists a $\Delta \boldsymbol{d} \geq 0$ that is consistent with the equilibrium changes in emissions, i.e. that satisfies $(\boldsymbol{u} - \boldsymbol{s}')^T \Delta \boldsymbol{d} = 0$, then this would imply a "free lunch" for the polluters whose emissions are covered by the scheme. We rule out free lunches.

Assumption 1 (No free lunch)**.** *There does not exist a non-negative change in emissions* $\Delta \boldsymbol{d} \geq 0$ *such that* $(\boldsymbol{u} - \boldsymbol{s}')^T \Delta \boldsymbol{d} = 0$.

Note that Assumption 1 is equivalent to a condition on the equilibrium gradient, namely $\boldsymbol{u} - \boldsymbol{s}' \neq \boldsymbol{0}$.

We are interested in the effects of policy-induced changes in demand. Recall that $\boldsymbol{\lambda} = (\lambda_i)_i$ denotes the vector of emissions policies. Let α_i denote the response of the equilibrium price p^* to a change in λ_i,

$$\boldsymbol{\alpha}_i \equiv dp^*/d\boldsymbol{\lambda}_i. \tag{3}$$

Similarly, let $\boldsymbol{\gamma}^i$ denote the change in the vector of equilibrium emissions in response to a change in λ_i,

$$\boldsymbol{\gamma}^i \equiv d\boldsymbol{d}^*/d\boldsymbol{\lambda}_i, \tag{4}$$

where $\boldsymbol{\gamma}^i_j$ denotes the change in demand in period j from a policy-induced demand change in period i. We adopt the notational convention that $\boldsymbol{x}_i$ (subscript i) denotes the i^{th} element of a vector $\boldsymbol{x}$, so a scalar, whereas $\boldsymbol{x}^i$ (superscript i) denotes th i^{th} column vector of a matrix $\boldsymbol{x}$. Hence, $\boldsymbol{x}^i_j$ the j^{th} element of the i^{th} vector of x.

We let Γ denote the matrix of all policy-induced changes in emissions so that, with a slight abuse of notation, we may write $\Gamma \boldsymbol{e}^i \equiv \boldsymbol{\gamma}^i$, where $\boldsymbol{e}^i$ is the unit vector with zeros everywhere but 1 at the i^{th} place (that is, e^i is the i^{th} column of the T-dimensional identity matrix).

Upon differentiating the equilibrium condition (2) with respect to $\boldsymbol{\lambda}_i$, one obtains a useful lemma.

Lemma 1. *All policy-induced changes in demand are orthogonal to the demand space gradient, or*

$$(\boldsymbol{u} - \boldsymbol{s}')^T \boldsymbol{\gamma}^i = 0, \tag{5}$$

for all i.

Because Lemma 1 holds true for all i, an immedate implication is that any linear combination of the set $\{\boldsymbol{\gamma}^i\}_i$ also satisfies the orthogonality property, and thus

$$(\boldsymbol{u} - \boldsymbol{s}')^T \Gamma = \boldsymbol{0}. \tag{6}$$

Equilibrium prices and demand respond intuitively to policy changes.

Lemma 2. *Prices increase with demand-increasing policies,*

$$\boldsymbol{\alpha} = -\frac{(\boldsymbol{u} - \boldsymbol{s}')^T}{(\boldsymbol{u} - \boldsymbol{s}')^T \boldsymbol{d}'} > 0, \tag{7}$$

and own-period demand increases, while other-period demand decreases:

$$\boldsymbol{\gamma}_j^i < 0 \quad \textit{for} \quad j \neq i, \tag{8}$$

$$\boldsymbol{\gamma}_i^i > 0. \tag{9}$$

A green paradox arises when an emissions-reducing policy in some period leads to an increase in aggregate equilibrium emissions.

Definition 3 (Green paradox)**.** *There is a green paradox if a demand-decreasing policy, $d\boldsymbol{\lambda} < 0$, causes an increase in aggregate emissions, $dD = d(\boldsymbol{u}^T \boldsymbol{d}^*) > 0$.*

The main result of the paper is that for any cap and trade scheme with a flexible emissions cap based on quantities, one can find a demand-reducing policy that induces a green paradox.

Theorem 1. *In every cap and trade system with a quantity-based flexible emissions cap and without a free lunch, there exists a policy $d\boldsymbol{\lambda} < 0$ that induces a green paradox, $d(\boldsymbol{u}^T \boldsymbol{d}^*) > 0$.*

Special cases of this result are well-established in the literature on quantity-based flexible emissions caps, see Gerlagh et al. (2021), Heijmans (2022), and Perino et al. (2022a,b) for discussions.

The problem that underlies Theorem 1 can be found in information economics. Economists motivate flexible emissions cap as a means through which the policymaker can incorporate polluting firms' private information in the setting of an emissions cap. The idea is that private knowledge about, for example, abatement costs can be inferred from equilibrium market behavior such as the choice of emissions in a particular period. Learning from this behavior, the policymaker can, it seems, make the policy more efficient by better aligning the cap on emissions with market fundamentals. This is the core argument underlying the many proposals for flexible (quantity-based) emissions caps, see for example Kollenberg and Taschini (2016, 2019), Lintunen and Kuusela (2018), Gerlagh and Heijmans (2020), Pizer and Prest (2020), and Karp and Traeger (2021).[5]

The problem with a quantity-based flexible cap is that the demand for emissions is an ambiguous signal of market fundamentals at best. The use of emissions allowances is not informative about the aggregate demand for emissions but only about relative demand, that is, the development of the demand for emissions over time. An increase in emissions today does not necessarily signal an increase in the demand for emissions overall; it merely indicates that the demand for emissions in the future, relative to demand today, has increased.

[5]Another, similar, argument is that an efficient emissions should be adjustable to accommodate unexpected shocks to the demand for emissions such as a change in macroeconomic conditions – and that these unexpected shocks, in turn, can be learned from observing market outcomes.

3 Discussion and Conclusions

We have shown that any emissions trading scheme in which the cap on emissions is updated in response to information on quantities necessarily suffers from a green paradox. In particular, it is always possible to implement a complementary climate policy aimed at reducing emissions that causes an increase in emissions overall through the endogenous emissions cap. This is a substantial generalization of recent results due to Gerlagh et al. (2021) and Osorio et al. (2021).

Perhaps it is useful to emphasize what this paper does *not* say. We do not argue that complementary emissions policies necessarily harm the environment. Rather, we prove that certain policies aimed at reducing emissions may harm the environment in some cases. Neither do we claim that complementary climate policies can never be combined with a cap and trade scheme. Our result only establishes that certain complementary climate policies increase emissions when complementing a cap and trade scheme which uses quantity information to update its cap. Relatedly, this paper does not say that an endogenous emissions cap is always a bad idea. Our Theorem specifically pertains to the use of quantity information to endogenize the emissions cap. If the policymaker were to use prices instead, our results cease to hold.

That final remark brings us to the policy implications of our work. There is no obvious advantage of using quantity information over allowance prices to update an emissions cap; however, this paper demonstrates that there are clear disadvantages. Unless a cap and trade scheme operates in absolute isolation from other climate policies – a claim hard to defend – it is a risky endeavor to combine emissions trading with complementary emissions policies. Given the enormous complexity of the climate problem, however, a combination of multiple and different policies is almost certainly called for. All in all, these observations would strongly favor the use of price signals over quantity signals to update an emissions cap.

Our policy implications also have policy relevance. Cap and trade schemes to regulate greenhouse gas emissions are used in most industrialized economies across the globe. Some of these schemes, including the Regional Greenhouse Gas Initiative (RGGI), California's ETS, and the ETS in Quebec, rely on allowance prices to update the cap on emissions. Our somewhat worrying result does not speak to those cap and trade schemes. That said, other – and important – cap and trade schemes do currently use quantity information to endogenize their caps. Examples include the European Union's Emissions trading scheme (the world's largest market for carbon), Switzerland's ETS, and South Korea's ETS.[6] The key policy takeaway of this paper is that complementary climate policies may be hard to combine with these cap and trade schemes. It is up to policymakers to formulate an appropriate policy-response to our findings. Perhaps the simplest possible strategy, though, is to abandon quantity-based cap adjustment and start using price signals instead. The examples in RGGI, California,

[6]South Korea does not formally use quantity information to update its cap, although it has historically done so according to the Asian Development Bank (2018).

and Quebec prove that to be possible.

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A Proofs

PROOF OF LEMMA 2

Proof. First diferentiate both sides of the equilibrium condition (2). This gives

$$-(\boldsymbol{u}-\boldsymbol{s}')^T\boldsymbol{d}'\boldsymbol{\alpha}_i = (\boldsymbol{u}-\boldsymbol{s}')^T\boldsymbol{\alpha}_i, \tag{10}$$

and we note that $-(\boldsymbol{u}-\boldsymbol{s}')^T\boldsymbol{d}' > 0$ by $\boldsymbol{d}' < 0$ and Assumption 1; (7) follows.

From (7), (8) immediately follows as $\boldsymbol{d}' < 0$ and $d\lambda_j = 0$ for $j \neq i$. Combined with Assumption 1, this implies (9). □

PROOF OF THE THEOREM

Proof. We prove the result by contradiction. Assuming there is no green paradox, we will show that, if so, one can construct a demand-reducing policy $d\boldsymbol{\lambda} < 0$ that decreases emissions in all periods $d\boldsymbol{d} < 0$. Since this is a direct contradiction of Assumption 1, that will conclude the proof.

We first observe that if there is no green paradox, then all policies that reduce demand in some period i decrease aggregate emissions. In this case, the matrix Γ is diagonally dominant over its columns and $\boldsymbol{u}^T\boldsymbol{\gamma}^i \geq 0$ for all i.

We next define normalized policies and equilibrium responses. Let $\boldsymbol{\kappa}^i \equiv d\boldsymbol{\lambda}^i/\boldsymbol{\gamma}^i_i < 0$ and $\boldsymbol{\eta}^i \equiv \boldsymbol{\gamma}^i/\boldsymbol{\gamma}^i_i$ so that if $\boldsymbol{\kappa}^i = -\boldsymbol{e}^i$, the policy reduces demand by one unit in period i. Let H be the matrix of normalized responses, $(H\boldsymbol{e}^i)_j = \boldsymbol{\eta}^i_j$. The matrix H is also diagonally dominant over its columns, a property it inherits from Γ, with ones on the diagonal and negative numbers everywhere else. In this notation, the effect of a policy vector $d\boldsymbol{\lambda} < 0$ on equilibrium emissions can be described through $d\boldsymbol{d} = H\boldsymbol{\kappa}$.

Choose the natural number A such that any policy which directly reduces demand in period i by one unit will reduce aggregate demand by at least A units, or $A = \min_i\{\boldsymbol{u}^T H\boldsymbol{e}^i\}$. Thus, A is the lower bound for the cumulative effectiveness of a policy in any period i. Because we assume that there is no green paradox, we have $A > 0$.

Recursively construct a series of vectors $\boldsymbol{z}^k$, with $k = 1, ..., \infty$, so that the series converges to $\boldsymbol{z}^k \to \boldsymbol{\kappa} < 0$, and $H\boldsymbol{\kappa} < 0$. We start for $k = 1$ with $\boldsymbol{z}^1 = -\boldsymbol{e}^1$. That is, the policy $\boldsymbol{z}^1$ decreases demand in the first period by one unit and increases demand in all other periods, but aggregate demand is decreased $\boldsymbol{u}^T H\boldsymbol{z}^1 < -A < 0$. This immediately implies that $\sum_i \max\{0, (H\boldsymbol{z}^1)_i\} < (1-A)$, i.e. the sum of all positive elements of H is bounded from above.

Assume that in step k, we know that (i) $\boldsymbol{u}^T H \boldsymbol{z}^k < 0$, and (ii) the sum of all positive elements is bound from above by $\sum_i \max\{0, (H\boldsymbol{z}^k)_i\} < (1-A)^k$. Given these two conditions, once may construct the next ($(k+1)$-th) element of the sequence in such a way that the properties (i) $\boldsymbol{u}^T H \boldsymbol{z}^{k+1} < 0$ and (ii) $\sum_i \max\{0, (H\boldsymbol{z}^{k+1})_i\} < (1-A)^{k+1}$ are transferred to the next inductive step. To see this, consider all positive elements of $\boldsymbol{u}^T H \boldsymbol{z}^k$, that we want to neutralize. Thus, let $\boldsymbol{z}^{k+1}$ be defined by $(\boldsymbol{z}^{k+1} - \boldsymbol{z}^k)_i = -\max\{0, (H\boldsymbol{z}^k)_i\} < 0$. The required properties follow immediately from this construction:

$$\boldsymbol{u}^T H \boldsymbol{z}^{k+1} = \boldsymbol{u}^T H \boldsymbol{z}^k + \boldsymbol{u}^T H (\boldsymbol{z}^{k+1} - \boldsymbol{z}^k) < 0 \tag{11}$$

$$\sum_i \max\{0, (H\boldsymbol{z}^{k+1})_i\} < (1-A) \sum_i \max\{0, (H\boldsymbol{z}^k)_i\} < (1-A)^{k+1} \tag{12}$$

Finally, we must show that the limit $\boldsymbol{\kappa}$ of $\boldsymbol{z}^k$ is in fact well-defined. This is easy. Note that, by construction, the sequence

$$\boldsymbol{u}^T (\boldsymbol{z}^{k+1} - \boldsymbol{z}^k) = \sum_i \max\{0, (H\boldsymbol{z}^k)_i\} < (1-A)^k \tag{13}$$

is a Cauchy sequence. To establish convergence it hence suffices to show that the sequence is defined on a compact set. To this end, recall that in any step κ, the sum of all positive elements of H was bound from above by $(1-A)^\kappa$. The aggregate increase in emissions is therefore never larger than $\sum_{\kappa=1}^{\infty} (1-A)^\kappa = 1/A < \infty$, where the last inequality follows from the assumption that $A > 0$. But this means the series of vectors $\boldsymbol{z}^k$ is defined on a closed and bounded set. By the Heine-Borel Theorem, a closed and bounded set is compact.

Having established convergence, we thus know that $\boldsymbol{z}^k \to \boldsymbol{\kappa} < 0$ and $H\boldsymbol{\kappa} < 0$. In non-normalized notation, we have therefore constructed a strict demand-reducing policy $d\boldsymbol{\lambda} < 0$ that implies a negative emissions response in all periods, $\boldsymbol{\gamma} = \Gamma d\boldsymbol{\lambda} < 0$. Combined with Assumption 1, $(\boldsymbol{u} - \boldsymbol{s}')^T \neq 0$, this implies $(\boldsymbol{u} - \boldsymbol{s}')^T \boldsymbol{\gamma} < 0$, which contradicts (5). □

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